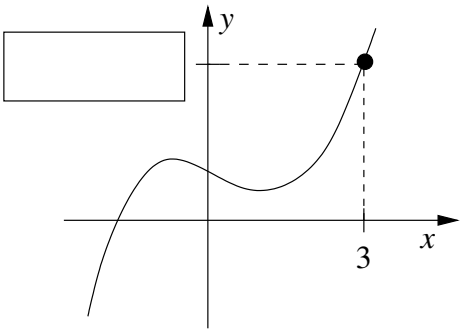
Sample Final Exam
Math 18A, Fall 2018

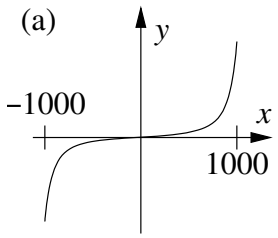
1. (6 points) Find two functions f and g such that $(g \circ f)(x) = \dfrac{1}{x^4 + 10}$ and neither $g(x) = x$ nor $f(x) = x$. No explanation necessary.

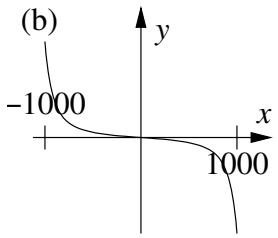
2. (6 points) Find all **real** solutions to the equation $x^2 - 5x - 7 = 0$. If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form (i.e., radicals and fractions, not decimals).

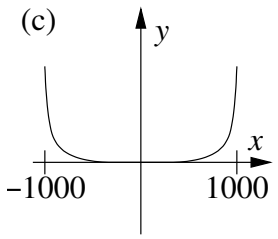
3. (6 points) The graph below is a sketch of the graph of the function $f(x) = x^3 - 5x + 7$ (not to scale). Fill in the missing value on the y-axis. Show all your work.

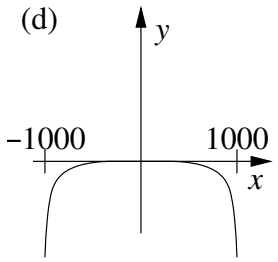


4. (6 points) Consider the function $f(x) = 4x^5 - 20x^2$. Which of the graphs below best matches the graph of $f(x)$? Note that the horizontal scale on the graph goes from $x = -1000$ to $x = 1000$, and the vertical scale is unspecified. Circle your answer, and briefly **EXPLAIN** why your answer is justified by the coefficients and powers appearing in $f(x)$.





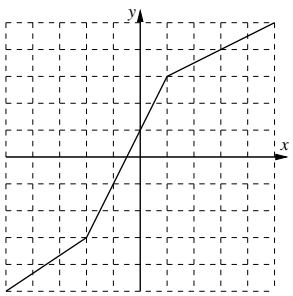




5. (6 points) Solve the inequality $1 < 2x - 6 < 3$. You may express your answer either in interval notation (e.g., "$[-16, 3] \cup (22, 55)$") or by inequalities (e.g., "$x > 325$").

6. (6 points) Find all **real** solutions to the equation $3x^2 - x - 4 = 0$. If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form (i.e., radicals and fractions, not decimals).

7. (6 points) Suppose f is a function whose graph is shown below. Find the value of $f^{-1}(1)$. No explanation necessary. You may assume that each square is 1 unit $\times$ 1 unit, that all of the pieces of the graph that look like straight lines are actually straight lines, and that points on the graph that appear to be very close to grid points actually land on those points.



8. (6 points) Solve the following system of linear equations. Show all your work.

$$\begin{aligned} x - y + 2z &= 8 \\ y - 4z &= -5 \\ z &= 3 \end{aligned}$$

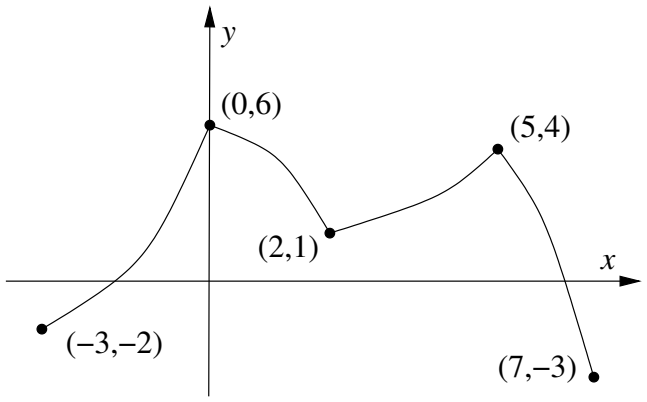
9. (8 points) Use long division to divide $f(x) = x^3 - 4x^2 + 5x - 3$ by $d(x) = x^2 + x - 4$. Show all your work, and express your final answer in the form $\dfrac{f(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

10. (8 points) Find the equation of the line through the points $(-5, 1)$ and $(6, 8)$. Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

11. (8 points) Draw the graph of $g(x) = \left(\dfrac{2}{3}\right)^x$. Clearly label any x-intercepts, y-intercepts, or asymptotes.

12. (8 points) Find all solutions to the equation $x - 3 - \dfrac{3}{x-5} = 0$. Show all your work, and leave your answer(s) in exact form (i.e., radicals and fractions, not decimals).

13. (8 points) Suppose $f(x)$ is a function whose graph is shown below (not to scale).



For the following, you may express interval answers either in interval notation (e.g., "on the intervals $(22, 55)$ and $[-16, 3]$") or by inequalities (e.g., "for $x > 325$").

(a) On which interval or intervals (values of x) is the function $f(x)$ **increasing**?

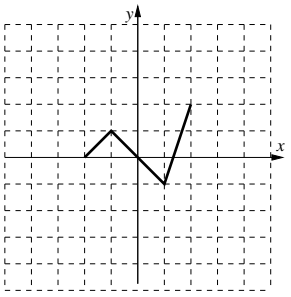
Interval(s) where $f(x)$ is decreasing:

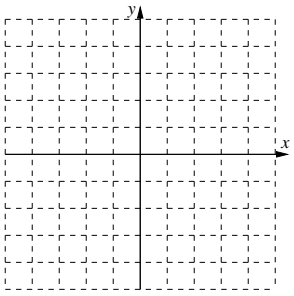
(b) At which **value(s) of** x does $f(x)$ have a **relative minimum**?

Value(s) of x where $f(x)$ has a relative minimum:

14. (8 points) Solve $\log_4(x + 9) = 3$ for x. Show all your work.

15. (8 points) Let $y = f(x)$ be the function whose graph is shown below left. (Each square is 1 unit $\times$ 1 unit.) On the axes below right, graph the function $y = f(x - 2) + 1$, paying careful attention to the vertical and horizontal scales. No explanation necessary.





16. (8 points) Expand the expression $\ln\left(\dfrac{(x-11)^5\sqrt{x^2+7}}{(x-3)^4}\right)$ as a sum, difference, and/or multiple of logarithms. Show all your work.

17. (10 points) Find all possible solutions to the following system:

$$\begin{aligned} 2x - 3y &= 3, \\ 6x + 5y &= 13. \end{aligned}$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

18. (10 points) Find the domain of the function $g(x) = \dfrac{x^2 - 7}{\sqrt{x^2 + 3x - 4}}$. Show all your work. You may express your answer either in interval notation (e.g., "$[-16, 3] \cup (22, 55)$") or by inequalities (e.g., "$x > 325$").

19. (10 points) Write the quadratic function $f(x) = 3x^2 - 12x - 21$ in standard form and sketch its graph. Label the vertex and the y-intercept of your graph. (You do not need to label the x-intercept(s), if any.)

20. (10 points) Let $f(x) = x^2 - 3x$. Simplify $\dfrac{f(5+h) - f(5)}{h}$ completely. Show all your work.

21. (12 points) The movie *Purple Panther Joins the Revengers* is now the highest-money-making superhero movie ever! It was more successful within the U.S. than overseas (outside the U.S.); in fact, *Purple Panther* made 35% more money in the U.S. than it did overseas.

If *Purple Panther* made a total of $1,652.3 million, combining both U.S. revenues and overseas revenues, how much money did *Purple Panther* make in the U.S.? **USE ALGEBRA**, not guessing, to determine the answer. Show all your work and give your final answer in the form of a complete sentence, using the correct units, rounding off the numerical part of your answer to the nearest .01 million dollars.

22. (12 points) Consider the polynomial function $f(x) = (x+1)(x-3)(x+4)(x-5)(x-11)$.

(a) List the real zero(s) of f. Zero(s):

(b) Sketch the graph of $f(x)$. In particular, make sure that the above information about zeros is clearly visible in your graph.

23. (12 points) Let

$$f(x) = x^3 + 2x^2 - 11x - 12.$$

Find the rational zeros of $f(x)$ by factoring $f(x)$ completely. Show all your work. Make sure you include both the complete list of zeros of $f(x)$ and the factorization of $f(x)$ in your final answer.

24. (12 points) The population of the state of Catatonia has been growing exponentially since it was founded at the beginning of the year 2000. If fact, if $P(t)$ is the population of Catatonia in millions of people, t years after its founding, then $P(t)$ can be modelled by

$$P(t) = 23.3e^{0.03t}.$$

If current trends continue, how many years after the beginning of the year 2000 will the population of Catatonia reach 53.7 million? Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.