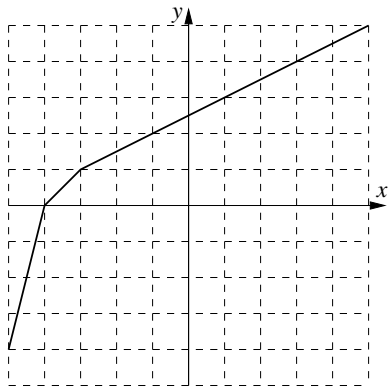


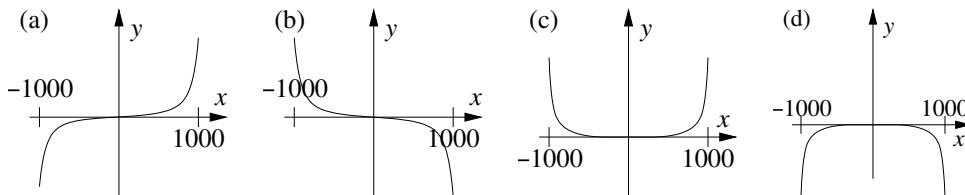
Sample Final Exam
Math 8, Spring 2017

This test consists of 24 questions on 4 pages (including this cover sheet), totalling 200 points. You are allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

1. (6 points) Find the y -intercept(s) of the graph of $x^2y - x^3 + y^2 = 7$. Show all your work, and leave your answer(s) in exact form.
2. (6 points) Solve the inequality $2 < 7x - 3 < 5$. You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).
3. (6 points) Suppose f is a function whose graph is shown below. Find the value of $f^{-1}(2)$. No explanation necessary. You may assume that each square is 1 unit \times 1 unit, that all of the pieces of the graph that look like straight lines are actually straight lines, and that points on the graph that appear to be very close to grid points actually land on those points.



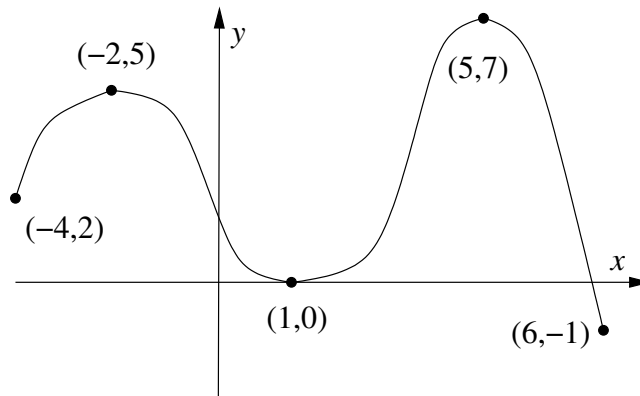
4. (6 points) Simplify $\log_3 \left(\frac{1}{81} \right)$. No explanation necessary.
5. (6 points) Consider the function $f(x) = -x^7 + 13x^4$. Which of the graphs below best matches the graph of $f(x)$? Note that the horizontal scale on the graph goes from $x = -1000$ to $x = 1000$, and the vertical scale is unspecified. Circle your answer, and briefly (1–3 sentences) **EXPLAIN** why the graph you chose is the best match.



6. (6 points) Solve the following system of linear equations. Show all your work.

$$\begin{aligned} x + 2y - 3z &= -5 \\ y - 5z &= 13 \\ z &= 2 \end{aligned}$$

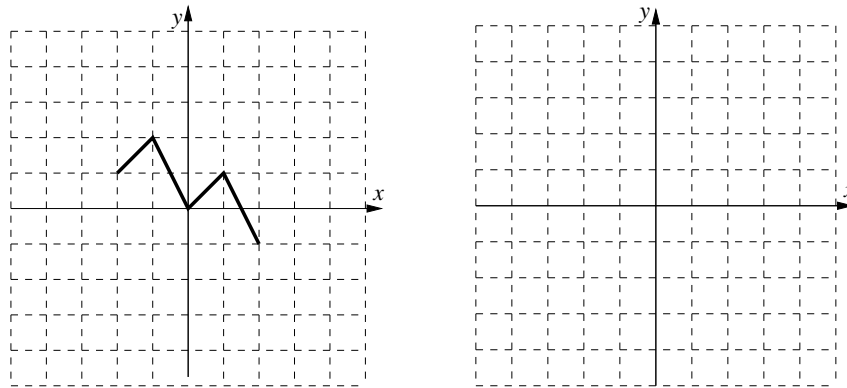
7. (6 points) Find all **real** solutions to the equation $x^2 - 7x + 5 = 0$. If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form.
8. (6 points) Find two functions f and g such that $(g \circ f)(x) = \ln(1 + x^4)$ and neither $g(x) = x$ nor $f(x) = x$. No explanation necessary.
9. (8 points) Find all solutions to the equation $\frac{2}{x-3} + x + 5 = \frac{11}{x-3}$. Show all your work, and leave your answer(s) in exact form.
10. (8 points) Expand the expression $\log\left(\frac{(x-3)^2\sqrt{x+5}}{(x+7)^3}\right)$ as a sum, difference, and/or multiple of logarithms. Show all your work.
11. (8 points) Use long division to divide $f(x) = 2x^3 - 7x^2 + 5x - 3$ by $d(x) = x^2 + x + 2$. Show all your work, and express your final answer in the form $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.
12. (8 points) Find the equation of the line through the points $(2, -7)$ and $(5, 13)$. Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)
13. (8 points) Suppose $f(x)$ is a function whose graph is shown below (not to scale).



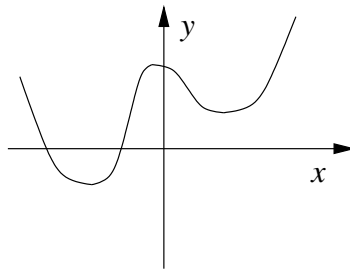
For the following, you may express interval answers either in interval notation (e.g., “on the intervals $(22, 55)$ and $[-16, 3]$ ”) or by inequalities (e.g., “for $x > 325$ ”).

- (a) On which interval or intervals (values of x) is the function $f(x)$ **increasing**?
- (b) At which **value(s) of x** does $f(x)$ have a **relative maximum**?

14. (8 points) Let $y = f(x)$ be the function whose graph is shown below left. (Each square is 1 unit \times 1 unit.) On the axes below right, graph the function $y = f(x - 2) - 1$, paying careful attention to the vertical and horizontal scales. No explanation necessary.



15. (8 points) Draw the graph of $g(x) = 5^x$. Clearly label any x -intercepts, y -intercepts, or asymptotes.
16. (8 points) Consider the following possible rough sketch of the graph of a polynomial function.



Which of the following polynomial functions most closely matches the above graph?

- (a) $p(x) = (x + 4)(x + 2)(x - 1)(x - 3)$ (b) $p(x) = (x + 4)(x + 2)(3x^2 + 1)$
 (c) $p(x) = (3x^2 + 1)(x - 1)(x - 3)$ (d) $p(x) = (3x^2 + 1)(x + 2)(x - 1)$

Clearly **CIRCLE** your answer, and below, use 1–2 sentences to **JUSTIFY** your answer, citing the specific features of the graph that distinguish your choice from the other choices.

17. (10 points) Find the domain of the function $g(x) = \frac{\sqrt{x^2 - 3x + 2}}{x - 12}$. Show all your work. You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).

18. (10 points) Let $f(x) = 2x^2 - 5$. Simplify $\frac{f(3 + h) - f(3)}{h}$ completely. Show all your work.

19. (10 points) Write the quadratic function $f(x) = x^2 - 6x - 4$ in standard form and sketch its graph. Label the vertex and the y -intercept of your graph. (You do not need to label the x -intercept(s), if any.)

20. (10 points) Find all possible solutions to the following system:

$$\begin{aligned}5x - 2y &= 7, \\2x - 3y &= 4.\end{aligned}$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

21. (12 points) The popularity of the children's card game *Poke Me, Man!* has exploded since 2010. To be precise, the number of *Poke Me, Man!* cards owned by American children can be modelled by the function

$$P(t) = 75000e^{.05t},$$

where $P(t)$ is the number of cards owned by American children t months after the beginning of 2010.

How many months passed after the beginning of 2010 before the number of *Poke Me, Man!* cards owned by American children reached 1,000,000 (one million)? Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

22. (12 points) Let

$$f(x) = x^3 - 7x^2 + 7x + 15.$$

Find the rational zeros of $f(x)$ by factoring $f(x)$ completely. Show all your work. Make sure you include both the complete list of zeros of $f(x)$ and the factorization of $f(x)$ in your final answer.

23. (12 points) Consider the polynomial function $f(x) = (x-1)(x+3)(x+4)(x-5)(x-6)$.

- List the real zero(s) of f .
- Sketch the graph of $f(x)$. In particular, make sure that the above information about zeros is clearly visible in your graph.

24. (12 points) On opening weekend, the blockbuster hit movie *Rouge One: A Makeup Wars Story* holds special midnight showings for charity. At these midnight showings, the movie sells a total of 103,874 tickets, some of which are for 3-D showings and the rest of which are for regular 2-D showings. The movie makes a total of \$1,598,051 in revenue, with regular tickets costing \$14 and 3-D tickets costing \$19.

How many people bought regular 2-D tickets, and how many people bought 3-D tickets? **USE ALGEBRA**, not guessing, to determine the answer. Show all your work and give your final answer in the form of a complete sentence, using the correct units.