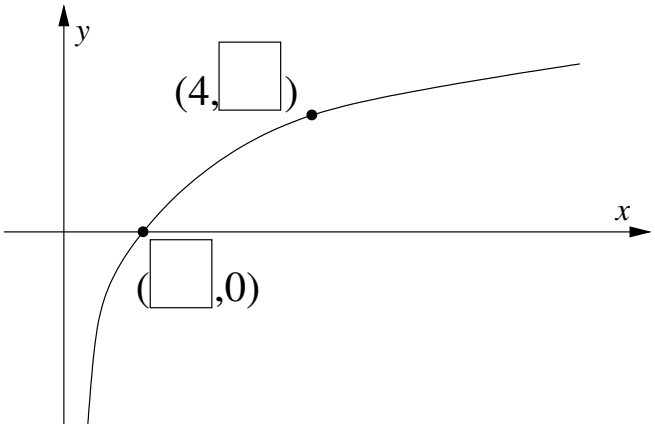
Sample Final Exam
Math 8, Spring 2015

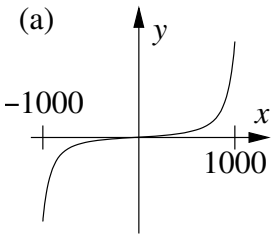
1. (6 points) Find all real solutions to $7x^2 - 3x + 5 = 0$, or if there are no real solutions, briefly **EXPLAIN** why there are no real solutions. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.).

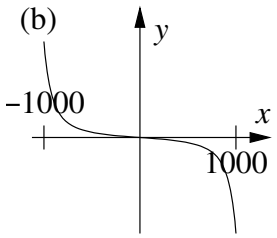
2. (6 points) Find two functions f and g such that $(f \circ g)(x) = \dfrac{1}{\sqrt{7 - 2x^2}}$ and neither $f(x) = x$ nor $g(x) = x$. No explanation necessary.

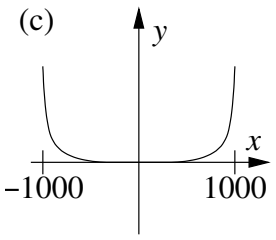
3. (6 points) The graph of $f(x) = \log_2(x)$ is sketched below (not to scale). Fill in the indicated x and y values on the graph. No explanation necessary.

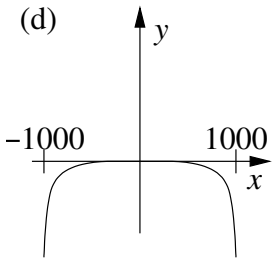


4. (6 points) Consider the function $f(x) = -7x^6 - 3x^2 + 2x - 15$. Which of the graphs below best matches the graph of $f(x)$? Note that the horizontal scale on the graph goes from $x = -1000$ to $x = 1000$, and the vertical scale is unspecified. Circle your answer, and briefly (1–3 sentences) **EXPLAIN** why the graph you chose is the best match.









5. (6 points) Use the Rational Zero Test to list all **possible** rational zeros of $3x^4 - 7x^3 + 12x^2 - 8$. Do **NOT** test these zeros to see if they are actually zeros of f. No explanation necssary.

6. (6 points) Solve the following system of linear equations. Show all your work.

$$\begin{aligned} x - y + 3z &= 1 \\ y + 2z &= 7 \\ z &= 3 \end{aligned}$$

7. (6 points) Find the x-intercept(s) of the graph of $y^2 + x^3y - x^2 + 3 = 0$. Show all your work.

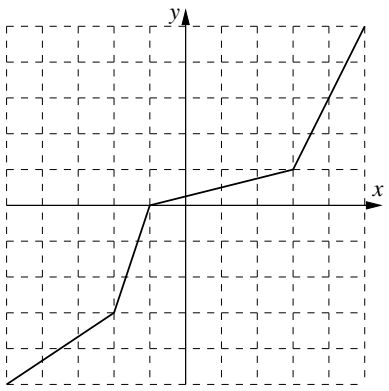
8. (6 points) Find all real solutions to $\dfrac{3}{x+2} - 6 = 5$, or if there are no real solutions, briefly **EXPLAIN** why there are no real solutions. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.).

9. (8 points) Let $f(x) = -5x^2 - 2x$. Simplify $f(7+h) - f(7)$ completely. Show all your work.

10. (8 points) Find the domain of the function $g(x) = \dfrac{\sqrt{25 - x^2}}{x - 3}$. You may express your answer either in interval notation (e.g., "on the intervals $(22, 55)$ and $[-16, 3]$") or by inequalities (e.g., "for $x > 325$"). Show all your work.

11. (8 points) Let $f(x) = x^2 - 7$ and $g(x) = \dfrac{5}{x+3}$. Find $(f \circ g)(x)$. Show all your work, and simplify your final answer.

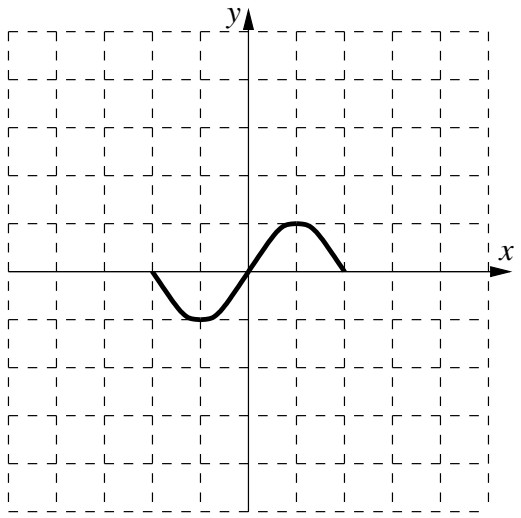
12. (8 points) Suppose $h(x)$ has the graph shown below, where each square is 1 unit by 1 unit. Find the value of $h^{-1}(-2)$, and circle or otherwise indicate the information on the graph that you are using to find the value of $h^{-1}(-2)$.

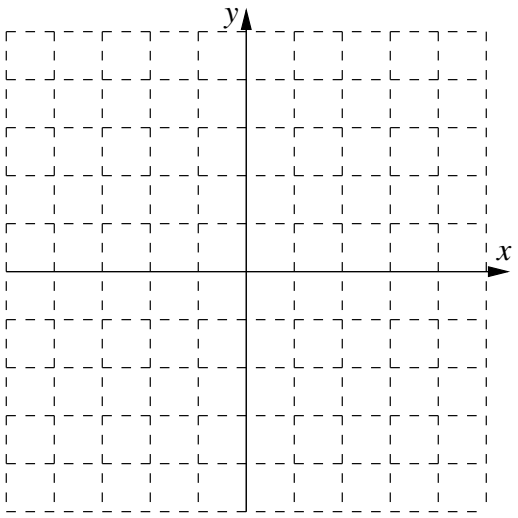


13. (8 points) Find the equation of the line through the points $(-5, 7)$ and $(4, 3)$. Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

14. (8 points) Find all real solutions to $\dfrac{2}{x+1} + \dfrac{3}{x+3} = 1$, or if there are no real solutions, briefly **EXPLAIN** why there are no real solutions. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.).

15. (8 points) Let $y = f(x)$ be the function whose graph is shown below left. (Each square is 1 unit $\times$ 1 unit.) On the axes below right, graph the function $y = 2f(x + 3)$, paying careful attention to the vertical and horizontal scales. No explanation necessary.





16. (8 points) Use the properties of logarithms to expand the expression $\log\left(\dfrac{\sqrt[3]{x+3}}{y^5 z}\right)$ as a sum, difference and/or multiple of logarithms. Assume all variables are positive. Show all your work. (There should be at least one middle step shown before the answer.)

17. (10 points) Find all possible solutions to the following system:

$$2x - 5y = 7,$$
$$3x + 4y = 2.$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.).

18. (10 points) Kawaii Games is looking at sales forecasts for its new video game *Death Kitty 2525*, and they determine that if they produce x units of the game, their profit (in dollars) will be $P(x) = -0.0002x^2 + 40x - 130000$. How many units should they produce to maximize their profit, and what will that maximum profit be?

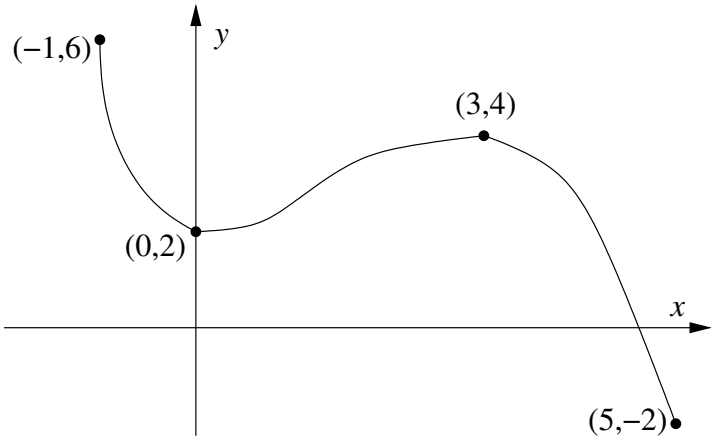
Show all your work, round off your final numerical answers to the nearest .01, if necessary, and give your final answers in the form of a complete sentence or sentences, using the correct units. Make sure that it is clear which answer is which.

19. (10 points)

(a) Draw the graph of $y = g(x) = 5^x$. Clearly label any x-intercepts, y-intercepts, or asymptotes.

(b) Find the domain and the range of $g(x)$. You may express your answers either in interval notation (e.g., "on the intervals $(22, 55)$ and $[-16, 3]$") or by inequalities (e.g., "$x > 325$" or "$-17 \leq y \leq 3$"). No explanation necessary.

20. (10 points) Suppose $f(x)$ is a function with the following graph:



(a) On which interval or intervals (values of x) is the function $f(x)$ **decreasing**?

(b) Find the **range** of $f(x)$.

For both parts of this problem, you may express your answer either in interval notation (e.g., "on the intervals $(22, 55)$ and $[-16, 3]$") or by inequalities (e.g., "$x > 325$" or "$-17 \leq y \leq 3$"). No explanation necessary.

21. (12 points) Mydd L'Manager works at SCP Advertising, and she is looking at how her salary has changed over the last two years. Between 2012 and 2013, her annual salary increased by \$11,000, and between 2013 and 2014, her annual salary increased by 10%. If Mydd's annual salary in 2014 is \$102,300, what was her annual salary in 2012?

Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

22. (12 points) Recall that the formula for the balance in an account that is compounded continuously is $A(t) = Pe^{rt}$, where P is principal and r is the annual interest rate. If you invest \$50,000 in an account at 4% interest, compounded continuously, how much time will it take for you to accumulate \$75,000 in your account?

Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

23. (12 points) Consider the polynomial function $f(x) = -2x^3 + 8x$.

(a) Factor $f(x)$ completely.

(b) List the real zero(s) of f.

(c) Sketch the graph of $f(x)$. In particular make sure that the above information about zeros is clearly visible in your graph.

24. (12 points) Let

$$f(x) = x^3 - x^2 - 8x + 12.$$

Find the rational zeros of $f(x)$ by factoring $f(x)$ completely. Show all your work. Make sure you include both the complete list of zeros of $f(x)$ and the factorization of $f(x)$ in your final answer.