

Sample Final Exam
Math 8, Fall 2014

This test consists of 24 questions on 4 pages (including this cover sheet), totalling 200 points. You are allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

This final did not cover Sections 5.1–5.3; otherwise, it conforms with the list of topics you have been given. However, you should expect that some of the types of questions seen here will not be on your final, and conversely, there will be types of questions on your final that are not found here. This sample is only meant to help you get used to the style and approximate length of the exam.

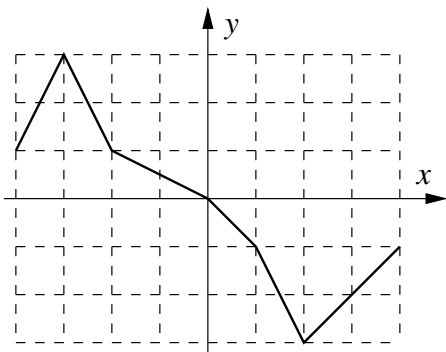
1. (6 points) Divide $x^3 - 3x^2 - x + 4$ by $x^2 + x - 1$. Show all your work, and write your final answer in the form $\frac{x^3 - 3x^2 - x + 4}{x^2 + x - 1} = q(x) + \frac{r(x)}{x^2 + x - 1}$, where $q(x)$ and $r(x)$ are polynomials.

2. (6 points) Find the real solution(s) of $\frac{2}{x+1} + \frac{3}{x+3} = 1$. Show all your work, and leave your answer(s) in exact form (fractional if necessary).

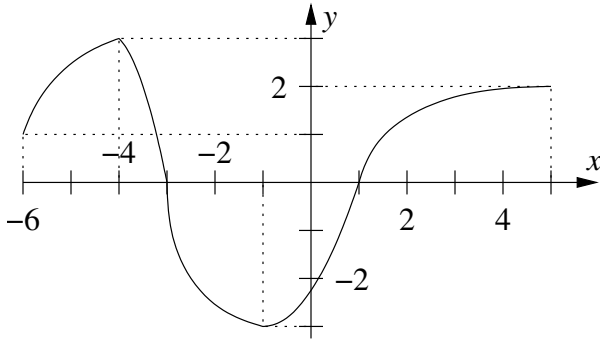
3. (6 points) Sketch the graph of $f(x) = \log_{10}(x - 4)$. Clearly indicate the x -intercept of the function on your graph.

4. (6 points) Find the real solution(s) of $\frac{2}{x-3} - 5 = 7$. Show all your work, and leave your answer(s) in exact form (fractional if necessary).

5. (6 points) Let $f(x) = x^2$, and let $g(x)$ be the function whose graph is shown below. (Each square is 1 unit \times 1 unit.) Evaluate $(g \circ f)(-2)$. Show all your work.

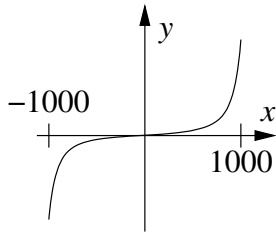


6. (6 points) Suppose the graph of a function $y = f(x)$ is shown below. On which interval or intervals (values of x) is the function **decreasing**? You may express your answer either in interval notation (e.g., “on the intervals $(22, 55)$ and $[-16, 3]$ ”) or by inequalities (e.g., “for $x > 325$ ”). No explanation necessary.



7. (6 points) Use the Rational Zero Test to list all **possible** rational zeros of $f(x) = 3x^4 - 2x^3 + 12x^2 - 5$. Do **NOT** test these zeros to see if they are actually zeros of f . No explanation necessary.
8. (6 points) Solve $2x^2 - 3x = 5$ by factoring. Show all your work, and leave your answer(s) in exact form, using fractions if necessary.
9. (8 points) Condense the expression $5 \ln(x - 4) + 2 \ln(x + 7) - 3 \ln(x - 2)$ to the logarithm of a single identity. Show all your work. (There should be at least one middle step shown before the answer.)
10. (8 points) Recall that the formula for the balance in an account that is compounded n times per year is $A = P \left(1 + \frac{r}{n}\right)^{nt}$. If you invest \$2,000 in an account at 6% interest, compounded 4 times per year, how much money will you have in the bank after 5 years? Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.
11. (8 points) Find the x -intercept(s) of the graph of $y = 2x^2 - 7x - 5$. Show all your work, and leave your answer(s) in exact form, using fractions or radicals if necessary.
12. (8 points) Solve $2 - 5x \leq 3 + 2x$. Show all your work. Leave numbers in exact form, using fractions if necessary, and express your answer either in interval notation (e.g., “on the intervals $(22, 55)$ and $[-16, 3]$ ”) or by inequalities (e.g., “for $x > 325$ ”).
13. (8 points) Find the equation of the line through the points $(1, 7)$ and $(5, -2)$. Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)
14. (8 points) Let $f(x) = 2x^2 - 7x$. Simplify $f(a + h)$ completely. Show all your work.

15. (8 points) Suppose we have a function $p(x)$ whose graph is sketched below. Note that the horizontal scale on the graph goes from $x = -1000$ to $x = 1000$, and the vertical scale is unspecified. Which of the formulas below best matches this graph? Circle your answer, and briefly (1–3 sentences) **EXPLAIN** which features of your chosen function match the most notable features of the graph.



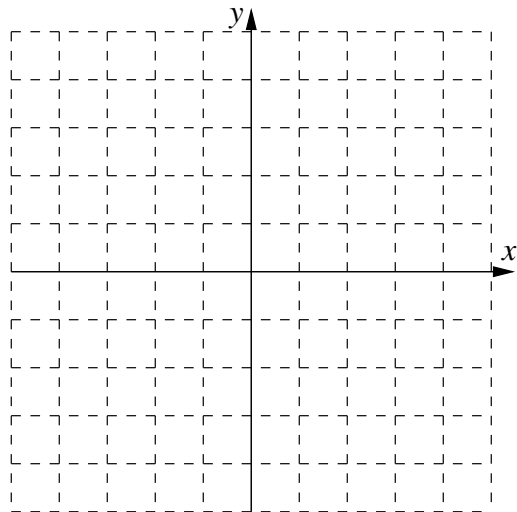
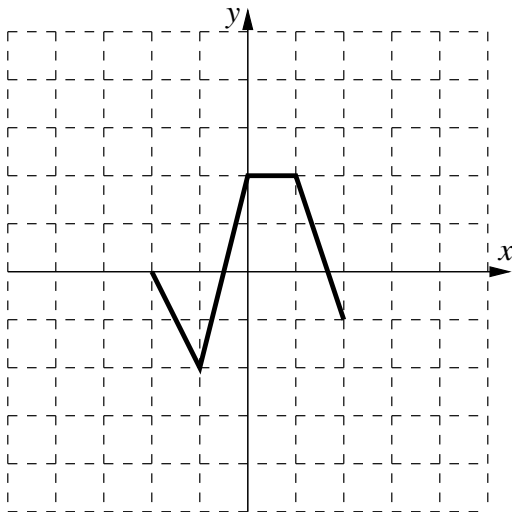
$$f_1(x) = 4x^4 - 2x + 7$$

$$f_2(x) = -3x^3 + 17x + 13$$

$$f_3(x) = -2x^6 + 21x^4 + 3x^2$$

$$f_4(x) = 7x^5 - 12x^4 + 3x + 10$$

16. (8 points) Let $y = f(x)$ be the function whose graph is shown below left. (Each square is 1 unit \times 1 unit.) On the axes below right, graph the function $y = f(x - 3) + 2$, paying careful attention to the vertical and horizontal scales. No explanation necessary.



17. (10 points) The ACME Roadbird Trap Company determines that if it produces x roadbird traps, the revenue it makes (in dollars) will be

$$R(x) = 1000x - 5x^2.$$

Suppose ACME wants to maximize its revenue.

- How many roadbird traps should ACME produce?
- What revenue (in dollars) will ACME make if it produces the optimal number of roadbird traps?

Show all your work. Round off your final numerical answers to the nearest .01 (if necessary), and give your final answer to each part of the problem in the form of a complete sentence, using the correct units. Finally, make it clear which is your answer to part (a) and which is your answer to part (b).

18. (10 points) Solve the equation $\frac{500}{1 + e^{-x}} = 470$ for x . Show all your work, and round off your final numerical answer to 3 (three) decimal places.

19. (10 points) For 2013, C. Orprat Tool, the CEO of Heartless Industries, Inc., raised his base salary by 20% over what it was in 2012. In fact, his company was doing so well that he gave himself an additional \$60,000 bonus in 2013, raising his total earnings for 2013 to a total (base salary plus bonus) of \$15,000,000.

What was Tool's base salary in 2012? Show all your work. Round off your final numerical answer to the nearest .01 (if necessary), and give your final answer in the form of a complete sentence, using the correct units.

20. (10 points) Find the domain of the function $g(x) = \frac{\sqrt{x+2}}{x-1}$. You may express your answer either in interval notation (e.g., "on the intervals $(22, 55)$ and $[-16, 3]$ ") or by inequalities (e.g., "for $x > 325$ "). Show all your work.

21. (12 points) Show that $f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$ are inverse functions by using the definition of inverse functions. Show all your work.

22. (12 points) Consider the polynomial function $f(x) = x^4 - x^2$.

(a) Factor $f(x)$ completely.

(b) List the real zeros of f .

(c) Sketch the graph of $f(x)$. In particular make sure that the above information about zeros is clearly visible in your graph.

23. (12 points) Let

$$f(x) = x^3 - 6x^2 + 3x + 10.$$

Find the rational zeros of $f(x)$ by factoring $f(x)$ completely. Show all your work. Make sure you include both the complete list of zeros of $f(x)$ and the factorization of $f(x)$ in your final answer.

24. (12 points) At midnight, 1000 of the cute, fuzzy, and rapidly multiplying alien creatures known as trebbles arrive at Station Deep Space π^2 . The trebbles multiply so rapidly that the number of trebbles on Deep Space π^2 is given by

$$P(t) = 1000e^{0.198t},$$

where t is the number of hours that have passed after the trebbles' arrival.

How many hours after the original arrival time will there be 60,000 trebbles on Deep Space π^2 ? Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.