Sample Final Exam Math 8, Spring 2014

This test consists of 24 questions on 4 pages (including this cover sheet), totalling 200 points. You are allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

Note that this final did not cover Section 4.6; otherwise, it conforms with the list of topics you have been given.

1. (6 points) Suppose the graph of a function y = f(x) is shown below. On which interval or intervals (values of x) is the function **increasing**? You may express your answer either in interval notation (e.g., "on the intervals (22,55) and (-16,3)") or by inequalities (e.g., "for x > 325"). No explanation necessary.



2. (6 points) Consider the graph of f(x) = |x|. Write an equation for the function whose graph is the same as the graph of f(x), but shifted to the **right** 3 units. No explanation necessary.

3. (6 points) **Circle** the polynomial function that best matches the graph below. Make sure your choice is clear; ambiguous or multiple answers will receive no credit.



$$f(x) = (x-1)^4 \qquad f(x) = (x+1)^4 \qquad f(x) = x^4 + 1 \qquad f(x) = x^4 - 1$$

$$f(x) = (x-1)^5 \qquad f(x) = (x+1)^5 \qquad f(x) = x^5 + 1 \qquad f(x) = x^5 - 1$$

4. (6 points) Evaluate the expression without using a calculator. Show all your work. $\log_2(8) =$ 5. (6 points) Suppose the graph of f is given below. Find the value of $f^{-1}(3)$. No explanation necessary.



6. (6 points) Use the Rational Zero Test to list all **possible** rational zeros of $f(x) = 2x^4 - 15x + 7$. Do **NOT** test these zeros to see if they are actually zeros of f. No explanation necessary.

7. (6 points) Let $f(x) = \sqrt[3]{x+5}$ and $g(x) = x^2 - 4$. Write down the formula of $(f \circ g)(x)$. Do **NOT** simplify your answer.

8. (6 points) Use the quadratic formula to solve the equation $5x = 13 - 3x^2$. Show all your work, and leave your answer in exact form (with radicals, **not** decimals). You do not need to simplify your answer.

9. (8 points) Divide $x^3 - 5x^2 + 2x - 7$ by $x^2 + 3x - 4$. Write your final answer in the form $\frac{x^3 - 5x^2 + 2x - 7}{x^2 + 3x - 4} = q(x) + \frac{r(x)}{x^2 + 3x - 4}$, where q(x) and r(x) are polynomials.

10. (8 points) Let $f(x) = 3x - x^2$. Simplify f(t+2) completely. Show all your work.

11. (8 points) Sketch the graph of $y = 7e^{0.2x}$. Label the coordinates of any intercepts, and label the coordinates of the point where x = 1.

12. (8 points) Find the x- and y-intercepts of the graph of $y^2 + 3xy - 2x = 36$. Show all your work.

13. (8 points) Find the equation of the line through the points (-2, 5) and (7, 1). Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

14. (8 points) Solve the equation $x - \frac{10}{x+1} = 2$. Show all your work.

15. (10 points) Consider the polynomial function $f(x) = x^3 - 4x = x(x+2)(x-2)$. (I.e., f(x) has already been factored for you.)

- (a) List the real zeros of f.
- (b) Does the graph of f(x) rise or fall to the left?

- (c) Does the graph of f(x) rise or fall to the right?
- (d) Sketch the graph of f(x). Make sure that the above information about zeros and right-hand and left-hand behavior is clearly visible in your graph.

16. (10 points) Katya deposits \$1000 in an interest-bearing account. The amount of money that will be in her account t years after she deposits the money is

$$A = 1000e^{0.05t}$$

How many years after Katya deposits her money will there be \$2500 in her account?

Show all your work. Round off your final numerical answer to the nearest .01, and give your final answer in the form of a complete sentence, using the correct units.

17. (10 points) Consider the quadratic function $f(x) = x^2 - 6x - 5$. Write f(x) in standard form and sketch its graph, labelling the vertex of the graph clearly. Show all your work.

18. (10 points) Use the properties of logarithms to expand the logarithmic expression $\ln\left(\frac{z^3}{13\sqrt{y}}\right)$ as a sum, difference and/or multiple of logarithms. Assume all variables are positive. Show all your work. (There should be at least one middle step shown before the answer.)

- **19.** (10 points) Let f(x) = 5x 7.
- (a) Find the inverse function $f^{-1}(x)$.
- (b) Verify that $f(f^{-1}(x)) = x$. (To save time, do **not** verify that $f^{-1}(f(x)) = x$.)

20. (10 points) Karla and Devina are reviewing the Marin Day Schools catering budget for 2012 and 2013. In 2013, MDS spent 10% more on catering than in 2012, and in the two-year period 2012–2013, MDS spent a total of \$11,550. How much did MDS spend on catering in 2012?

Show all your work. Round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

21. (10 points) Find the domain of the expression $\sqrt{x^2 - 4x - 5}$. Show all your work. You may express your answer either in interval notation (e.g., "on the intervals (22,55) and (-16,3)") or by inequalities (e.g., "x > 325"). Make it clear whether endpoints of intervals either are or are not included in the domain.

22. (10 points) Let $g(x) = 1 + \sqrt{x+2}$.

- (a) Describe the sequence of transformations from $f(x) = \sqrt{x}$ to g.
- (b) Sketch the graph of g.

23. (12 points) The rock band Jennifer and the Jennifers is building a rectangular stage on which they will perform their new album *Pelican*. A diagram of the stage is shown below (not to scale); the shaded area indicates the area in which they can actually play, as they must stay at least 1 meter away from the edge of the stage. If the (outside) length of the stage is twice the (outside) width of the stage, and the playing area (the shaded area) is 40 square meters, what is the (outside) width of the stage?

- (a) Set up a quadratic equation to find the (outside) width of the stage.
- (b) Solve your equation and find the (outside) width of the stage. Show all your work. Give your final answer in the form of a complete sentence, using the correct units.



24. (12 points) Let

$$f(x) = x^3 - 4x^2 + x + 6.$$

Find the rational zeros of f(x) by factoring f(x) completely. Show all your work. Make sure you include both the complete list of zeros of f(x) and the factorization of f(x) in your final answer.