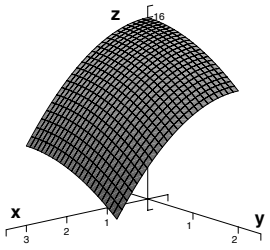
Sample Exam 3
Math 32, Fall 2015

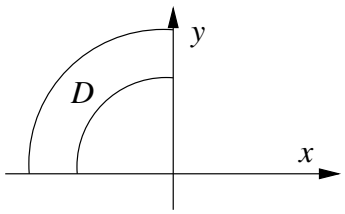
1. (18 points) Let R be the region described by $2 \le x \le 3$, $4 \le y \le 7$. Compute the double integral

$$\iint_R (5x + 13y)^{1/2}\, dA.$$

No explanation necessary, but show all your work. **DO NOT SIMPLIFY** your final numerical answer.

2. (16 points) Let E be the 3-dimensional region bounded by $z = 0$, $z = 16 - x^2 - y^2$, $x = 0$, $x = 3$, $y = 0$, and $y = 2$. (The appropriate portion of the graph of $z = 16 - x^2 - y^2$ is shown below.) Express the triple integral $\iiint_E f(x, y, z)\, dV$ as an iterated integral. **DO NOT EVALUATE THIS INTEGRAL.**





3. (26 points) Let D be a flat plate in the portion of the xy-plane with $x \le 0$ and $y \ge 0$ (i.e., the 2nd quadrant of the xy-plane) that is bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$, as shown above.

Suppose that the density of D at (x, y) is $\rho(x, y) = y$.

(a) Find the mass of the flat plate D. No explanation necessary, but show all your work. **DO NOT SIMPLIFY** your final numerical answer.

(b) Write down an interated integral that can be used to find $\overline{x}$, the average x value, or x coordinate of the center of mass, for the flat plate D. **DO NOT EVALUATE THIS INTEGRAL.**

4. (20 points) Let D be the region bounded by $y = 2x$, $x = 0$, and $y = 6$.

(a) Sketch the region D.

(b) Calculate the double integral

$$\iint_D 2xe^{y^3}\, dA$$

by expressing the double integral as an iterated integral where you **integrate x first** (i.e., in the order $dx\, dy$). Show all your work, and **DO NOT SIMPLIFY** your final numerical answer.

5. (20 points) Let $f(x,y) = 2x^2 - x^2y - \dfrac{2y^3}{3} + 11y^2 + 4$. Calculations show that the derivatives of f are:

$$f_x(x,y) = 4x - 2xy, \qquad f_y(x,y) = -x^2 - 2y^2 + 22y,$$
$$f_{xx}(x,y) = 4 - 2y, \qquad f_{xy}(x,y) = -2x, \qquad f_{yy}(x,y) = 22 - 4y.$$

(In other words, you are given these derivatives and you do not need to calculate them yourself.)

(a) It turns out that $(6,2)$ is a critical point of f. Explain, using the definition of critical point, how you can check that $(6,2)$ is a critical point of f.

(b) In fact, $(0,0)$, $(0,11)$, and $(6,2)$ are all critical points of f (i.e., you are now given this). Classify each of these critical points as a local minimum, local maximum, or saddle point. Show all your work.