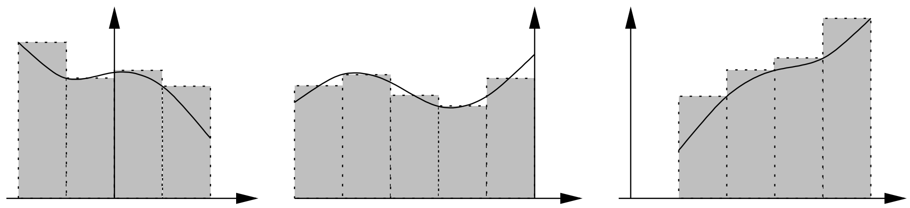
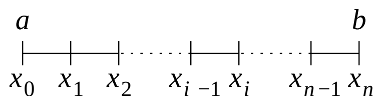
Math 31

A pictorial guide to the definition of the definite integral

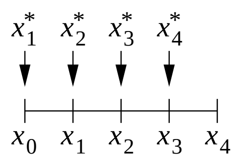


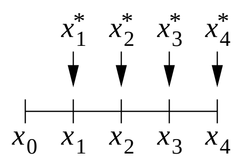
For each positive integer n:

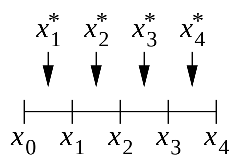
1. We subdivide the interval $[a, b]$ into n subintervals $[x_{i-1}, x_i]$ of width $\Delta x = \dfrac{b-a}{n}$:



2. For each subinterval $[x_{i-1}, x_i]$, we choose a point x_i^* inside that subinterval by some method, such as (here shown with $n = 4$):

   - The left-hand endpoint method: 

   - The right-hand endpoint method: 

   - The midpoint method: 

3. We then form the *Riemann sum* $\displaystyle\sum_{i=1}^{n} f(x_i^*)\Delta x = f(x_1^*)\Delta x + \cdots + f(x_n^*)\Delta x$.

   For the left hand and right hand endpoint methods, this becomes:

   - Left-hand sum: $f(x_0)\Delta x + \cdots + f(x_{n-1})\Delta x$.
   - Right-hand sum: $f(x_1)\Delta x + \cdots + f(x_n)\Delta x$.

Finally, we define the definite integral of f from $x = a$ to $x = b$ by taking the limit:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$