A car is being tested for braking distance. Before the test starts, it is travelling at 30 mph (44 ft/sec), and then at time $t = 0.0$ sec, the brakes are applied. The following data is then collected from the speedometer of the car.

t (sec)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$v(t)$ (ft/sec)	44	43	41	38	34	30	26	21	15	8	0

The eventual goal here is to find a good approximation of the total distance the car travels between $t = 0.0$ and $t = 1.0$ (the braking distance). Assume that the speed of the car is always decreasing.

1. If you assume that the car travels at the speed $v(0)$ (i.e., at its initial speed) from $t = 0.0$ to $t = 0.2$, how far does the car travel in that time?

2. On the graph on page 2, draw a rectangle whose area is the answer to part 1.

3. If you assume the car travels at the speed $v(0)$ from $t = 0.0$ to $t = 0.1$, how far does the car travel in that time?

4. On the graph on page 3, draw a rectangle whose area is the answer to part 3.

5. What is the car's highest speed between $t = 0.0$ and $t = 0.2$, and at what time is this achieved?

6. What is the car's lowest speed between $t = 0.0$ and $t = 0.2$, and at what time is this achieved?

7. Using your answers to 1, 5, and 6, give upper and lower bounds for how far the car travels between $t = 0.0$ and $t = 0.2$.

8. Repeat 7 in the intervals $0.2 \leq t \leq 0.4$, $0.4 \leq t \leq 0.6$, $0.6 \leq t \leq 0.8$, $0.8 \leq t \leq 1.0$.

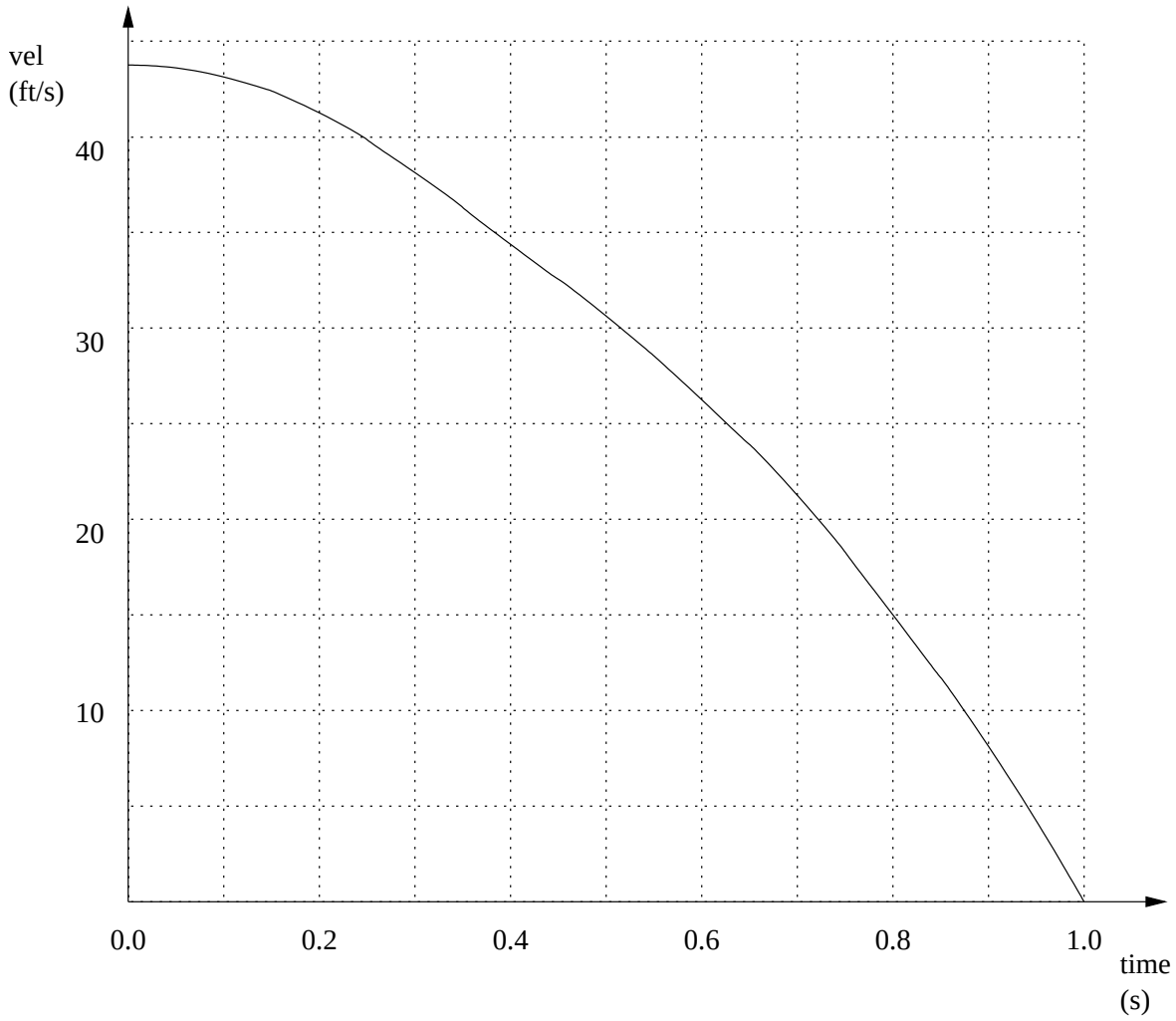
9. Using your answers to 7 and 8, give upper and lower bounds for the distance the car travels between $t = 0.0$ and $t = 1.0$. (Also, see graph on page 2.)

10. Repeat 7–9, but use the intervals $0.0 \leq t \leq 0.1$, $0.1 \leq t \leq 0.2$, etc. How does the difference between your upper and lower bounds here compare to the difference between your upper and lower bounds in 9? (Also, see graph on page 3.)

11. If the data were available, and you used the intervals $0.0 \leq t \leq 0.05$, $0.05 \leq t \leq 0.10$, and so on, to estimate the braking distance, what would the difference between the upper and lower bounds be?

12. Again, if the data were available, what do you think would happen if you considered even smaller intervals? If Δt is the interval size, and there are n intervals between 0.0 and 1.0, can you write out a formula for the upper estimate? For the lower estimate? What do you think happens as $n \to \infty$, making $\Delta t \to 0$?

Illustrate your answer to 9 on the graph below.



Illustrate 10 on the graph below.