Sample exam 1
Math 30, Fall 2018

This is Exam 1 from the last time I taught Math 30. Please treat this sample not as a guide to what will be covered in our exam, or the length of the exam but as a guide to what the questions will be like. (In particular, there will be material from the homework on solving polynomial, rational, trig, and exponential equations that was not on this exam.)

You will be allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

1. (12 points) The internet cat video *Kittens Playing with Jars!* is released at midnight. The table below shows the total number of times the video had been viewed as a function of t, where t is the number of hours after the video is released.

t (hrs)	1.0	1.2	1.4	1.6
Total views	25,927	29,325	32,214	35,798

(a) Calculate the slope of the secant line between the points $t = 1.0$ and $t = 1.2$.

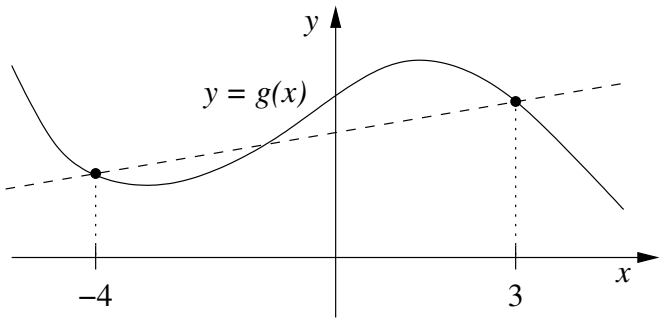
(b) Same, but for $t = 1.2$ and $t = 1.4$.

(c) Estimate the slope of the tangent line at $t = 1.2$. (This represents the rate, in views per hour, at which the video is being viewed at that time.)

Show all your work, and make sure you use the correct units for each answer.

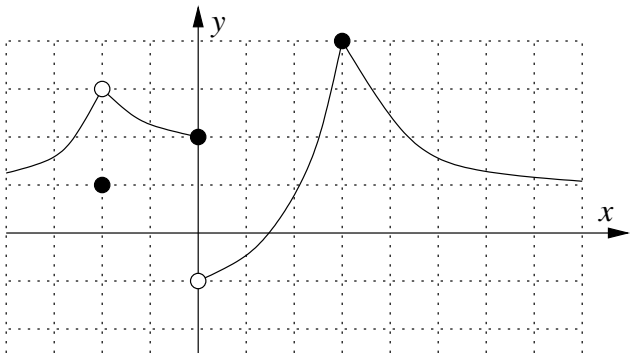
2. (12 points) Guess the value of the limit $\lim_{x \to 5} \dfrac{\ln x - \ln 5}{x - 5}$ by evaluating $f(x) = \dfrac{\ln x - \ln 5}{x - 5}$ at $x = 4.99, 4.999, 5.01, 5.001$, correct to six decimal places. Show all your work.

3. (12 points) Let g be a function whose graph is shown below (solid curve, not to scale). Find the **equation** of the dashed line. Show all your work; you do not need to simplify your answer. Your answer may be in terms of $g(x)$ (i.e., something like g(whatever) may appear in a correct answer).



4. (10 points) Solve $7e^{5x-3} = 11$ for x. Show all your work, and leave your answers in **exact form**, i.e., in terms of e, ln, and so on. You do not need to simplify your final answer.

5. (12 points) Let h be a function whose graph is shown below. (Each square is 1×1.)



For each of the following limits, if the limit exists, determine its value, based on the above graph; and if the limit does not exist, write "DNE". No explanation necessary.

$$\lim_{x\to -2} h(x) = \qquad\qquad \lim_{x\to 0^+} h(x) =$$

$$\lim_{x\to 3} h(x) = \qquad\qquad \lim_{x\to +\infty} h(x) =$$

6. (14 points) Evaluate the limit $\lim_{h\to 0} \dfrac{(4+h)^3 - 64}{h}$, using the algebraic limit laws. Show all your work.

7. (14 points) Sketch the graph of a function $f(x)$ such that:

- $\lim_{x\to 2^-} f(x) = 3$ and $\lim_{x\to 2^+} f(x) = 7$;
- f is continuous from the left at $x = 2$;
- $\lim_{x\to -3} f(x) = 6$;
- f is not continuous at $x = -3$; and
- f is continuous at all values of x other than $x = 2$ and $x = -3$.

No explanation necessary. Make sure you pay careful attention to the horizontal and vertical scales of your graph.

8. (14 points) Suppose we know that the function $f(x)$ has the properties

$$\lim_{x\to 3^+} f(x) = 7, \qquad\qquad \lim_{x\to 3^-} f(x) = 5.$$

(a) Briefly (1–2 sentences) **EXPLAIN** what it means to say that $\lim_{x\to 3^-} f(x) = 5$, without using the word "limit". (In other words, you are supposed to explain the idea of limit, using this particular example.)

(b) If $\lim_{x\to 3^+} f(x) = 7$ and $\lim_{x\to 3^-} f(x) = 5$, is it possible for f to be continuous at $x = 3$? If the answer is YES, sketch the graph of a possible $f(x)$ that has those two properties and is also continuous at $x = 3$; if the answer is NO, briefly (1–2 sentences) **EXPLAIN** why it is not possible for f to be continuous at $x = 3$.