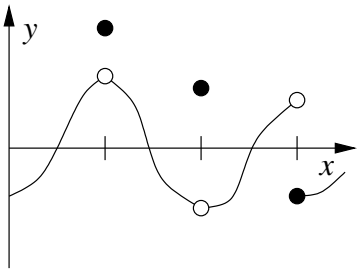
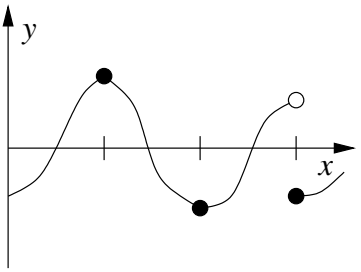
Class prep quiz on sections 2.5–2.6, Stewart's Calculus (8th ed.)

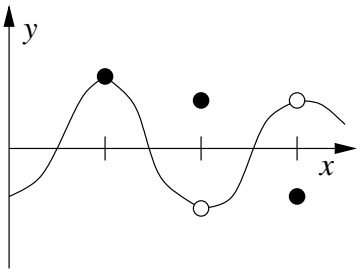
1. Suppose $h(x)$ is a function such that

   - h is continuous at $x = 1$;
   - $\lim_{x\to 2} h(x)$ exists, but h is not continuous at $x = 2$; and
   - $\lim_{x\to 3} h(x)$ does not exist.

   Which of the following could be the graph of h?

(a)


(b)


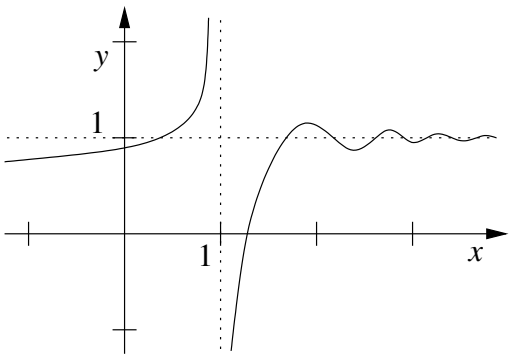
(c)


(d) None of the above.

2. Which of the following statements **must always** be true?
   (a) If $f(a)$ exists, then f is continuous at $x = a$.
   (b) If f is continuous at $x = a$, then $\lim_{x\to a} f(x)$ exists.
   (c) If $\lim_{x\to a} f(x)$ exists, then f is continuous at $x = a$.
   (d) If $\lim_{x\to a} f(x)$ exists, then $f(a)$ exists.

3. Suppose g is a continuous function. Which of the following statements is **not** a consequence of the Intermediate Value Theorem?
   (a) If $g(x)$ is never equal to 0, then it is impossible to have $g(-4) = 7$ and $g(13) = -1$.
   (b) If $g(-1) = 7$ and $g(5) = 12$, then $g(x) \neq 0$ for $-1 \leq x \leq 5$.
   (c) If $g(-5) = -13$ and $g(-2) = 4$, then $g(x) = 0$ for some x between -5 and -2.
   (d) If $g(3) = 7$ and $g(13) = -5$, then $g(x) = 0$ for some x between 3 and 13.

4. Suppose the graph of $y = h(x)$ looks like:



Which of the following statements seems most likely to be correct?

(a) $\lim_{x\to\infty} h(x)$ exists and $\lim_{x\to 1} h(x) = +\infty$.

(b) $\lim_{x\to\infty} h(x)$ does not exist and $\lim_{x\to 1} h(x) = +\infty$.

(c) $\lim_{x\to\infty} h(x)$ exists and $\lim_{x\to 1} h(x)$ does not exist.

(d) $\lim_{x\to\infty} h(x)$ does not exist and $\lim_{x\to 1} h(x)$ does not exist.