## Math 221A, problem set 07 Due: Mon Nov 01 Last revision due: Mon Dec 06

Problems to be turned in: Problem x.y.z of Artin denotes problem y.z in Chapter x.

- 1. Artin 7.2.2.
- 2. Enumerate the conjugacy classes of  $S_4$ , and for each conjugacy class C:
  - Find the number of elements in  $\mathcal{C}$ .
  - Identify the isomorphism type of the centralizer of an element of  $\mathcal{C}$ .
  - Verify the orbit-stabilizer theorem for the action of  $S_4$  on  $\mathcal{C}$  by conjugation.
- 3. Artin 7.2.14.
- 4. Artin 7.3.3. (You may assume Artin 7.3.2.)
- 5. Artin 7.4.4.
- 6. Artin 7.4.8.
- 7. Assume  $n \geq 5$ .
  - (a) Let N and K be normal subgroups of a group G. Prove that  $N \cap K$  is normal in G, and therefore, normal in both N and K.
  - (b) Prove that if N is a normal subgroup of  $S_n$ , then N is either 1,  $A_n$ , or  $S_n$ . (Suggestion: Consider  $N \cap A_n$ .)