

**Math 221A, problem set 06**  
**Due: Mon Oct 18**  
**Last revision due: Mon Dec 06**

**Problems to be turned in:** Problem x.y.z of Artin denotes problem y.z in Chapter x.

1. Let  $G$  be the symmetry group of a cube centered at the origin, including orientation-reversing isometries like the matrix  $-I$ .
  - (a) List all elements in the stabilizer of a face of the cube. What does the Orbit-Stabilizer equation look like for the orbit and stabilizer of a face?
  - (b) Same, but for an edge of the cube.
  - (c) Same, but for a vertex of the cube.
2. Artin 6.7.8.
3. Let  $H$  and  $K$  be subgroups of a group  $G$  (possibly infinite), and suppose that  $H$  has finite index in  $G$ .
  - (a) Use group actions to prove that  $[K : H \cap K] \leq [G : H]$ .
  - (b) Give an example of  $H, K \leq G$  such that  $[K : H \cap K]$  does not divide  $[G : H]$ .
4. Artin 6.9.3.
5. Classify all transitive group actions of  $D_4 = \langle x, y \mid 1 = x^2 = y^4, xyx^{-1} = y^{-1} \rangle$  on four objects.
6. Let  $G$  be a group (possibly infinite), and suppose that  $H$  is a subgroup of  $G$  such that  $[G : H] = n$ . Prove that  $G$  has a normal subgroup of index dividing  $n!$  ( $n$  factorial).
7. Artin 6.12.5.