Math 221A, problem set 05 Due: Mon Oct 11 Last revision due: Mon Dec 06

Problems to be turned in: Problem x.y.z of Artin denotes problem y.z in Chapter x.

- 1. Artin 6.3.2. Note that this is only true for isometries of \mathbf{R}^2 .
- 2. Artin 6.3.4.
- 3. Let $L = \langle a, b \rangle$ be a lattice in \mathbb{R}^2 . Note that we write the operation in L as +.
 - (a) Prove that every sublattice of index n in L is the kernel of a surjective homomorphism $\varphi: L \to G$, where G is an abelian group of order n.
 - (b) List all sublattices of index 4 in L in terms of their generators. (For example, $\langle 4a, b \rangle$ is a sublattice of index 4 in L.) For each such sublattice L', draw a picture of L' inside L. (You can draw L as the standard unit lattice $L = \langle e_1, e_2 \rangle$.)
- 4. Artin 6.5.6.
- 5. Artin 6.5.8.