Math 221A, problem set 01
Due: Mon Aug 30
Last revision due: Mon Sep 20

Problems to be turned in: Problem x.y.z of Artin denotes problem y.z in Chapter x.

1. Let $\alpha = (1\ 7\ 4)(2\ 5\ 3\ 9\ 10\ 8\ 6\ 12)$ and $\beta = (2\ 10\ 11\ 5\ 3\ 12\ 4\ 7\ 8\ 6\ 9)$ be elements of S_{12}.
   (a) Compute $\alpha\beta$ and α^{-1}, in cycle form.
   (b) Find the orders of α, α^{-1}, β, and $\alpha\beta$.

2. The *cycle shape* of $\alpha \in S_n$ is the set (or actually, multiset) of the lengths of the cycles obtained when α is expressed as a product of disjoint cycles.

   For several $\alpha, \beta \in S_6$, compare α, β, and $\alpha\beta\alpha^{-1}$. Form a conjecture about cycle shapes. (You do not need to prove your conjecture.)

3. Artin 2.6.7.

4. Artin 2.4.3.

5. (modified Artin 2.4.4) Describe all groups G that contain no nontrivial proper subgroups. (Here, "describe" means state a theorem and prove it.)

6. Artin 2.4.8(b).

7. Fix $n \geq 3$.
   (a) Suppose $a, b, c, d \in \{1, \ldots, n\}$ with $a < b$ and $c < d$. Prove that $(a\ b)(c\ d)$ is a product of 3-cycles. (Try cases.)
   (b) Prove that every even permutation in S_n is a product of 3-cycles, or in other words, that A_n is generated by the set of 3-cycles in S_n.