

Sample Exam 2
Math 19, Spring 2013

1. (6 points) Calculate $\frac{3-2i}{7+i}$. Show all your work, and write your final answer in the form $a+bi$, where a and b are real numbers.

2. (12 points) Consider the quadratic function $f(x) = 2x^2 - 11x + 4$.

(a) Express $f(x)$ in standard form.

(b) Find the vertex and the x - and y -intercepts of $f(x)$.

(c) Sketch the graph of $f(x)$, labelling the vertex and intercepts clearly.

Show all your work, and express all answers in **exact** form, i.e., if an answer involves square roots, etc., do not convert the square roots to decimal form.

3. (14 points) Lyrion “(1/3)-man” Cannister of Easteros is buying swords for his army. He calculates that to buy x swords, it will cost him $f(x) = 4.5x + 2$ gold pieces.

(a) Find a formula for f^{-1} . **EXPLAIN** what f^{-1} represents, in terms of swords and gold pieces, in **ONE SENTENCE**.

(b) Find $f^{-1}(137)$. **EXPLAIN** what your answer tells you about swords and gold pieces in **ONE SENTENCE**.

4. (12 points) Let $r(x) = \frac{3(x-2)(x+7)}{(x-3)(x+5)} = \frac{3x^2 + 15x - 42}{x^2 + 2x - 15}$.

(a) Find the x -intercept(s) of the graph of $r(x)$, if any.

(b) Find the y -intercept(s) of the graph of $r(x)$, if any.

(c) Find the vertical asymptote(s) of $r(x)$, if any.

(d) Find the horizontal asymptotes(s) of $r(x)$, if any.

Show all your work, and clearly indicate which answer is which.

5. (6 points) Give an example (i.e., make up an example) of a polynomial function $y = P(x)$ of degree $n > 1$ such that as $x \rightarrow +\infty$, $y \rightarrow -\infty$, and as $x \rightarrow -\infty$, $y \rightarrow +\infty$. No explanation necessary.

6. (12 points) Let $P(x) = x^4 - 3x^3 + 7x^2 - 10x - 1$ and $D(x) = x^2 + x - 2$. Use long division to divide $P(x)$ by $D(x)$, and express the quotient $P(x)/D(x)$ in the form $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$. Show all your work.

7. (6 points) Suppose $P(x)$ is a polynomial with real coefficients. Is it possible that the zeros of $P(x)$ are precisely $x = -3$, $x = 5$, $x = 2 + 5i$, and no others? **EXPLAIN** your answer in **ONE SENTENCE**.

8. (16 points) Let $P(x) = x^4 - 4x^3 - x^2 + 16x - 12$. Find all zeros of $P(x)$, and factor $P(x)$ completely. Show all your work.

9. (16 points) Let

$$f(x) = (-5)(x + 1000)^2(x - 7)^2(x + 999)^3(x - 6) = -5x^8 + \dots$$

- (a) Describe the end behavior of $f(x)$. In particular, describe what happens to $y = f(x)$ as $x \rightarrow +\infty$ and $x \rightarrow -\infty$.
- (b) Find the **sign** of the y -intercept of $f(x)$.
- (c) Find the zeros of $f(x)$ and the multiplicity of each zero.
- (d) For each zero of $y = f(x)$, determine the behavior of the graph $y = f(x)$ near that zero; in particular, determine if the graph cuts through the x -axis, bumps the x -axis, or slides through the x -axis.
- (e) Sketch the graph of $y = f(x)$, making sure that all of the above information is indicated clearly. (It is more important that the key features are visible than for your graph to be drawn to scale.)