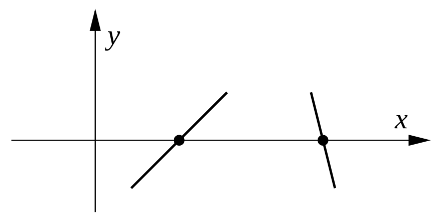
Behaviors of polynomials at zeros: Cut, bump, slide
Math 19

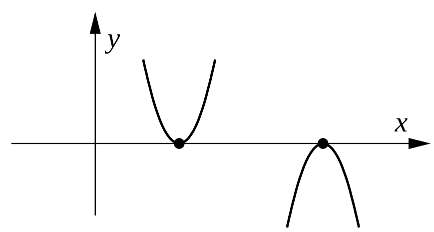
Suppose $f(x)$ is a polynomial function, and suppose r is a zero of $y = f(x)$ (i.e., suppose $f(r) = 0$). For the portion of $y = f(x)$ near $x = r$, there are three things that can happen.

If r is a zero of multiplicity 1, then for some constant C, $f(x) \approx C(x - r)$ for x near r, and the graph $y = f(x)$ will **cut** the x-axis at $x = r$ and **cross** over to the other side:



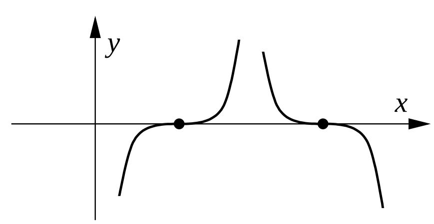
Note that this is like the behavior of $y = x$ or $y = -x$ at $x = 0$.

If r is a zero of multiplicity 2, then for some constant C, $f(x) \approx C(x - r)^2$ for x near r, and the graph $y = f(x)$ will **bump** (or **touch**) the x-axis at $x = r$ and stay on the same side:



Note that this is like the behavior of $y = x^2$ or $y = -x^2$ at $x = 0$.

If r is a zero of multiplicity 3, then for some constant C, $f(x) \approx C(x - r)^3$ for x near r, and the graph $y = f(x)$ will **slide** through the x-axis at $x = r$ and **cross** over to the other side:



Note that this is like the behavior of $y = x^3$ or $y = -x^3$ at $x = 0$.

Zeros of multiplicity 4, 6, 8, etc., act like zeros of multiplicity 2, replacing $(x - r)^2$ with $(x-r)^4$, $(x-r)^6$, $(x-r)^8$, etc. Zeros of multiplicity 5, 7, 9, etc., act like zeros of multiplicity 3, replacing $(x - r)^3$ with $(x - r)^5$, $(x - r)^7$, $(x - r)^9$, etc.