Math 142, problem set 09
REVISED TUE NOV 15
Outline due: Thu Nov 10
Final version due: Wed Nov 16

Problems to be turned in:

1. Solve the recurrence relation $a_n = 3a_{n-1} - a_{n-2} - a_{n-3}$ with initial conditions $a_0 = 4$, $a_1 = 2$, $a_2 = 7$.

2. Let a_n be the number of tilings of an $n \times 1$ grid using red and blue dominoes (2×1 tiles) and black, white, and green single squares (1×1 tiles).

   (a) Find a recurrence relation for a_n and use that recurrence relation and the values of a_1 and a_2 to find a closed formula for a_n.

   (b) Find constants C and α such that a_n is $C\alpha^n$ rounded off to the nearest integer (except possibly for finitely many values of n).

3. An *octal* sequence is a sequence in the digits 0–7. Let a_n be the number of octal sequences of length n not containing 1234 as a subsequence.

   (a) Find a recurrence relation for a_n.

   (b) Find the solutions to the characteristic equation of the recurrence relation. (Use a computer algebra system like Wolfram Alpha and give your answer in terms of decimal approximations.)

   (c) Find the initial values $a_1, \ldots, a_4$.

   (d) Set up, but do **not** solve, a system of linear equations whose solution gives you a closed form for a_n.

4. Solve the recurrence relation $a_n = 3a_{n-1} + 2n$, $a_0 = 1$.

5. Solve the recurrence relation $a_n = 2a_{n-1} + n^2$, $a_0 = 0$.

6. Suppose you start off a job with an annual salary of \$80,000, and each year you get a 3% cost-of-living salary increase and a \$1,000 merit salary increase. Let s_n be your annual salary n years after you start your job, e.g., $s_0 = 80000$.

   (a) Find a recurrence relation for s_n.

   (b) Solve your recurrence relation to find a closed formula for s_n.