

Math 142, problem set 04
Outline due: Wed Sep 28
Final version due: Mon Oct 03

For all of the following problems, explain/justify your answer, and write your final numerical answer as a sum or product of factorials, $\binom{n}{k}$, and so on.

Problems to be turned in:

1. Seventeen (17) 5th graders from Peabody Elementary are visiting Roosevelt Middle School to see what it's like, and each 5th grader is paired with a middle school buddy to show them around. Choosing from among these 34 students:

- (a) How many ways are there to choose a 5th grader and a middle school student who are *not* assigned to be buddies?
- (b) How many ways are there to choose two 5th graders and three middle school students, *none* of whom are assigned to be buddies?

2. There is an old nursery rhyme that goes something like this:

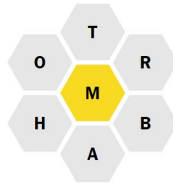
As I was going to St. Ives,
I met a man with seven wives;
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits:
Man, kits, cats, sacks, and wives,
How many were going to St. Ives?

The classic trick answer is “one, the narrator.” However, your question is: How many were coming *from* St. Ives?

3. Suppose you have a pair of 12-sided dice, one blue and one gold.
 - (a) How many possible outcomes are there when you roll both dice?
 - (b) What is the probability that the number on the blue die will be (strictly) less than the number on the gold die?
 - (c) Now suppose you roll your pair of dice twice. What is the probability that the total on the first roll will be equal to the total on the second roll?
4. How many sequences of 13 decimal digits are there with no consecutive pair of digits the same? (Example: 0101234567890.)

(continued on next page)

5. The *New York Times* publishes a daily puzzle called the Spelling Bee, where you are given a grid like this:



and you have to form as many ordinary English words as you can, at least four letters long, using the letters in the grid as many times as you like and using the center letter at least once. For example, for the above grid, *BOTTOM* and *MATH* are acceptable words, but *RAM* (too short) and *BROTH* (no *M*) are not.

Suppose we ignore the constraint that Spelling Bee words must be words in English (e.g., *BRHATM* and *MMMMTTT* are now acceptable). In that case, for any given Spelling Bee puzzle, how many 8-letter words are there?

6. Going back to the 34 students from problem 1:
- (a) How many ways are there for those 34 students to stand in line?
 - (b) If we add the extra condition that no two students from the same school can stand next to each other, how many ways are there for them to stand in line?
7. Suppose you have 18 cards, 11 spades (all different) and 7 hearts (all different).
- (a) How many ways are there to deal 5 cards from among the cards that you have?
 - (b) If you deal 5 cards, what is the probability that 2 of them will be spades and 3 of them will be hearts?
 - (c) If you deal 5 cards, what is the probability that you will deal more spades than hearts?