Math 131A, problem set 09
Outline due: Thu Apr 18
Completed version due: Tue Apr 23
Last revision due: TBA

Problems to be done but not turned in: 29.1, 29.3, 29.5, 29.7, 29.9, 29.11, 29.13, 29.15, 29.17, 32.1, 32.3, 32.5, 32.7, 33.1, 33.3, 33.7, 33.9, 33.11, 33.13.

Problems to be turned in: All numbers refer to exercises in Ross.

1. Prove (iii) of Corollary 29.7.
2. Let I be an interval, $a \in I$. Suppose $f : I \to \mathbf{R}$ is differentiable on I, $f'(a) > 0$, and $f'(x)$ is continuous at a.
   (a) Prove that there exists some $\delta > 0$ such that f is strictly increasing on $[a-\delta, a+\delta]$.
   (b) Explain why part (a) does not contradict PS08 #7.
3. Suppose $f, g : \mathbf{R} \to \mathbf{R}$ are differentiable, that $f(0) = g(0)$, and that $f'(x) > g'(x) + 2$ for $x > 0$. Prove that $f(x) > g(x) + 2x$ for any $x > 0$.
4. Let $f : (0, \infty) \to \mathbf{R}$ be differentiable, and suppose that both f and f' are strictly increasing on $(0, \infty)$.
   (a) Use the Mean Value Theorem to prove that $f'(a) > 0$ for some $a \in (1, 2)$.
   (b) Let $m = f'(a)$, where a is taken from part (a). Prove that for $x > 2$, we have $\dfrac{f(x) - f(2)}{x - 2} \geq m$.
   (c) Prove that $\lim_{x \to \infty} f(x) = +\infty$. (Suggestion: Use (b) to get a lower bound for $f(x)$.)
5. Integration notes, Problem 1.2.
6. Integration notes, Problem 2.5.
7. Integration notes, Problem 2.6.