Math 131A, problem set 02
Outline due: Wed Feb 07
Completed version due: Mon Feb 12
Last revision due: Thu Mar 28

Problems to be done but not turned in: 5.1, 5.3, 5.5, 5.7, 7.1, 7.3, 7.5.
Problems to be turned in: All numbers refer to exercises in Ross.

1. Let S be a bounded subset of $\mathbf{R}$, and let $T = \{x - y \mid x, y \in S\}$.
   (a) Prove that $\sup S - \inf S$ is an upper bound for T.
   (b) Prove that $\sup T = \sup S - \inf S$. (Suggestion: Arbitrarily Close Criterion.)

2. We define an *interval* in $\mathbf{R}$ to be a nonempty subset $S \subseteq \mathbf{R}$ such that if $x, y \in S$ and $x < z < y$, then $z \in S$.
   (a) Give an example of a nonempty $T \subseteq \mathbf{R}$ that is *not* an interval, and prove that T is not an interval.
   (b) Now suppose S is a *bounded* interval, let $a = \inf S$, and let $b = \sup S$. (Note that a and b need not be elements of S.) Prove that if $a < x < b$, then $x \in S$.

3. Let S be a nonempty set of *negative* real numbers such that $\inf S = -\infty$, and let $T = \left\{ \frac{1}{x} \,\middle|\, x \in S \right\}$. Prove that $\sup T = 0$.

4. Ex. 7.2.

5. Ex. 7.4.

6. Guess the value of $\lim \dfrac{2n - 15}{n^2 + 5}$, and prove your answer, using the definition of limit.

7. Guess the value of $\lim \dfrac{3n - 5}{4n + 7}$, and prove your answer, using the definition of limit.