Sample Exam 1
Math 129B, Spring 2012

This exam covered through section 3.3 of the text, and not 3.4. Otherwise, it covers roughly the same material as ours will.

1. (16 points) Let V be a vector space, let $\mathbf{v}_1, \dots, \mathbf{v}_k$ be vectors in V, and let S be a subspace of V. Define what it means for $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ to span S.

2. (14 points) Let X be a set. Define the vector space $\mathbb{F}(X)$ (a vector space of functions). In particular, explain what the "vectors" in $\mathbb{F}(X)$ are, and define vector addition and scalar multiplication in $\mathbb{F}(X)$.

For questions 3–5, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer as specifically as possible. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (12 points) Let

$$S = \left\{(x, y, z) \in \mathbb{R}^3 \mid |x| = |y|\right\}.$$

Then S is a subspace of $\mathbb{R}^3$.

4. (12 points) Let X, Y, Z be elements of $\mathbb{M}(2,2)$ such that $aX + bY + cZ = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ if $a = b = c = 0$. Then $\{X, Y, Z\}$ is linearly independent.

5. (12 points) Let $V = \{(v_1, v_2) \mid v_1, v_2 \in \mathbb{R}\}$, and let addition and scalar multiplication in V be defined by:

$$(v_1, v_2) + (w_1, w_2) = (v_1 + w_1, v_2 + w_2),$$
$$r(v_1, v_2) = (v_1, v_2).$$

Then V is a vector space.

6. (16 points) **PROOF QUESTION.** Let V be a vector space.

(a) For $r \in \mathbb{R}$ and $\mathbf{v} \in V$, in **one sentence**, explain the difference between $(-r)\mathbf{v}$ and $-(r\mathbf{v})$.

(b) Prove that if $r \in \mathbb{R}$, $\mathbf{v} \in V$, then $(-r)\mathbf{v} = -(r\mathbf{v})$, relying only on the axioms of a vector space and the theorem that $0\mathbf{v} = \mathbf{0}$ for all $\mathbf{v} \in V$. Make sure you cite which axiom or theorem you are using each time you use one.

7. (18 points) **PROOF QUESTION.** Let A and B be fixed (constant) 2×2 matrices, and let W be the set of all 2×2 matrices X such that $AX = XB$. In other words, let

$$W = \{X \in \mathbb{M}(2,2) \mid AX = XB\}, \; A, B \in \mathbb{M}(2,2) \text{ fixed.}$$

Prove that W is a subspace of $\mathbb{M}(2,2)$.