Math 129B, problem set 03
Outline due: Wed Feb 15
Due: Mon Feb 27
Last revision due: Wed Mar 14

Problems to be done, but not turned in: (3.1) 7, 11; (3.2) 6, 9, 13; (3.3) 5, 7, 9, 10, 13, 17.

Problems to be turned in:

1. Let V be a vector space, let W_1 and W_2 be subspaces of V, and let

$$U = \{\mathbf{v} \in V \mid \mathbf{v} = \mathbf{w}_1 + \mathbf{w}_2 \text{ for some } \mathbf{w}_1 \in W_1,\ \mathbf{w}_2 \in W_2\}.$$

   Prove that U is a subspace of V.

2. (3.2) 11(b).

3. (3.2) 14.

4. Let V be a vector space, and let $\mathbf{v}, \mathbf{w}, \mathbf{x}$ be vectors in V such that $\mathbf{v} + \mathbf{w} + \mathbf{x} = \mathbf{0}$. Let $W_1 = \operatorname{span}\{\mathbf{v}, \mathbf{w}\}$, and let $W_2 = \operatorname{span}\{\mathbf{w}, \mathbf{x}\}$. Must it be true that $W_1 = W_2$? Prove or give a counterexample.

5. (3.3) 15.

6. Let V be a vector space, and let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be nonzero vectors in V.
   (a) Give an example of a vector space V and $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in V$ such that $\mathbf{v}_3 \in \operatorname{span}\{\mathbf{v}_1 - \mathbf{v}_2, \mathbf{v}_2 - \mathbf{v}_3\}$.
   (b) Now suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent. Is it possible that $\mathbf{v}_3 \in \operatorname{span}\{\mathbf{v}_1 - \mathbf{v}_2, \mathbf{v}_2 - \mathbf{v}_3\}$? Give an example or prove that it is not possible.