Definition of a vector space: Operations

A **vector space** is a set V with

- addition ($\mathbf{v} + \mathbf{w} \in V$ defined for $\mathbf{v}, \mathbf{w} \in V$) and
- scalar multiplication ($r\mathbf{v} \in V$ defined for $r \in \mathbb{R}$, $\mathbf{v} \in V$)

such that for all $\mathbf{v}, \mathbf{w}, \mathbf{x} \in V$, $r, s \in \mathbb{R}$, the following axioms are satisfied:

Definition of a vector space: Axioms

1. (Commutativity) $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$.
2. (Associativity) $(\mathbf{v} + \mathbf{w}) + \mathbf{x} = \mathbf{v} + (\mathbf{w} + \mathbf{x})$.
3. (Additive identity) There exists a vector in V, called $\mathbf{0}$, such that $\mathbf{v} + \mathbf{0} = \mathbf{v}$.
4. (Additive inverse) For each $\mathbf{v} \in V$, there exists some $-\mathbf{v} \in V$ such that $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$.
5. (Distributivity) $r(\mathbf{v} + \mathbf{w}) = r\mathbf{v} + r\mathbf{w}$.
6. (Distributivity) $(r + s)\mathbf{v} = r\mathbf{v} + s\mathbf{v}$.
7. (Associativity of scalar multiplication) $r(s\mathbf{v}) = (rs)\mathbf{v}$.
8. (Scalar identity) $1\mathbf{v} = \mathbf{v}$.