Sample exam 3, Spring 2002

1. (10 points) Let A be an $n \times n$ matrix. Define what it means for $\mathbf{v}$ to be an eigenvector of A.

2. (12 points) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 2 & 1 \\ 3 & 6 & c & 3 \\ 4 & 8 & 12 & 10 \end{bmatrix}$. Find the value of $\det A$, in terms of c, and determine all values of c for which A is invertible. Show all your work.

3. (12 points) Choosing A, and performing a calculation, we see that

$$A = \begin{bmatrix} 2 & -1 & -5 & 2 & 10 \\ 1 & -1 & -4 & 3 & 4 \\ 1 & -2 & -7 & 2 & 7 \\ 1 & 0 & -1 & 1 & 4 \end{bmatrix}, \qquad \operatorname{rref}(A) = \begin{bmatrix} 1 & 0 & -1 & 0 & 5 \\ 0 & 1 & 3 & 0 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for $\operatorname{Col} A$. No explanation necessary, but show all your work.

(b) Find a basis for $\operatorname{Null} A$. No explanation necessary, but show all your work.

4. (14 points) Let $A = \begin{bmatrix} 3 & 8 & 8 & 0 \\ -3 & -4 & 0 & 3 \\ 3 & 3 & -1 & -3 \\ -2 & 2 & 8 & 5 \end{bmatrix}$. You may take it as given that the characteristic polynomial of A is $(t+1)^2(t-2)(t-3)$. (In other words, do not spend time checking this fact.)

Exactly one of the eigenspaces of A has dimension 2. Find the eigenvalue λ of this eigenspace, and find a basis for this eigenspace. Show all your work.

5. (8 points) (T/F) Let V be the set of all $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in $\mathcal{R}^4$ such that $x_1 - x_2 + x_3 + 2x_4 = 0$ and $5x_2 - 3x_3 + x_4 = 0$. Then V is a subspace of $\mathcal{R}^4$.

6. (8 points) (T/F) Let V be the set of all $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in $\mathcal{R}^2$ such that $|x_1| = |x_2|$. Then V is a subspace of $\mathcal{R}^2$.

7. (8 points) (T/F) It is possible that there exists a subspace V of $\mathcal{R}^5$, and vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5$ in V, such that $\dim V = 4$ and $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$ is a basis for V.

8. (8 points) (T/F) Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be nonzero vectors in $\mathcal{R}^4$, and let $V = \operatorname{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$. Then $\dim V$ must be 3.

9. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for a subspace V of $\mathcal{R}^5$.

(a) (10 points) Is $\{3\mathbf{v}_1, 2\mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2\}$ a basis for V? Explain why or why not.

(b) (10 points) Is $\{\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_2 - \mathbf{v}_3, -\mathbf{v}_3\}$ a basis for V? Explain why or why not.