Sample exam 2, Spring 2002

1. Let $T : \mathcal{R}^n \to \mathcal{R}^m$ be a function.

(a) (8 points) Define what it means for T to be linear (that is, define what it means for T to be a linear transformation).

(b) (6 points) Define what it means for T to be onto.

2. (14 points) Let $A = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & 8 \\ -1 & 0 & -3 \end{bmatrix}$, and let $B = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 1 & 2 \end{bmatrix}$.
If A is invertible, compute A^{-1}, and verify, by direct computation, that $AA^{-1} = I_3$. If A is not invertible, explain how you know that A is not invertible, and compute AB. Either way, show all your work.

3. (18 points) Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6$ be vectors in $\mathcal{R}^4$, and let A be the 4×6 matrix whose columns are $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6$. Furthermore, suppose that

$$\text{rref}(A) = \begin{bmatrix} 1 & -2 & 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Does $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6\}$ span $\mathcal{R}^4$? Briefly **JUSTIFY** your answer.

(b) Is $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6\}$ linearly independent?
If yes, briefly **EXPLAIN** why.
If no, briefly **EXPLAIN** why not, and express one of the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6$ as a linear combination of the others.

4. (8 points) (T/F) Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ be vectors in $\mathcal{R}^7$. It is possible that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathcal{R}^7$.

5. (8 points) (T/F) Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ be vectors in $\mathcal{R}^2$ such that $\mathbf{u}_1 \neq \mathbf{u}_2$, $\mathbf{u}_1 \neq \mathbf{u}_3$, $\mathbf{u}_2 \neq \mathbf{u}_3$. Then $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ *must* span $\mathcal{R}^2$.

6. (8 points) (T/F) If A and B are 2×2 matrices, then $AB = BA$.

7. (8 points) (T/F) Let A be a 4×4 matrix. If A is invertible, then A is the product of 4×4 elementary matrices.

8. (8 points) (T/F) Let A be a 3×3 invertible matrix. It is possible that there exists a 3×3 matrix $B \neq \mathbf{0}$ such that $BA = \mathbf{0}$. (In both instances, $\mathbf{0}$ is the 3×3 zero matrix.)

The last question on this exam was a proof; there will be no proofs in our exam.