Sample exam 1, Spring 2002

1. Let $\mathbf{u}_1, \ldots, \mathbf{u}_k$ be vectors in $\mathcal{R}^n$.

(a) (8 points) Define what it means to be a linear combination of $\mathbf{u}_1, \ldots, \mathbf{u}_k$.

(b) (8 points) Define the span of $\mathbf{u}_1, \ldots, \mathbf{u}_k$.

2. For each of the the following calculations, performed the calculation if it is defined, or explain why the calculation is not defined. Show all your work.

(a) (6 points) $\begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ -6 \end{bmatrix}$

(b) (6 points) $2 \begin{bmatrix} 2 & -1 & 0 \\ 0 & -3 & 1 \end{bmatrix}^T - 3 \begin{bmatrix} 5 & -2 \\ 3 & 1 \\ 0 & -2 \end{bmatrix}$

For questions 3–4, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit. If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (10 points) Every consistent system of 3 linear equations in 3 variables has a unique solution (i.e., exactly one solution).

4. (10 points) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$. If $\mathbf{x}$ is a vector such that $A\mathbf{x} = \mathbf{0}$, then $\mathbf{x}$ is an element of $\mathcal{R}^3$.

5. (16 points) Suppose that the following matrix is the reduced row-echelon form (RREF) of the augmented matrix of a system of linear equations in x_1, x_2, x_3, x_4, x_5 (**FIVE** variables):

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -4 & -5 \\ 0 & 0 & 1 & 0 & -3 & 2 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{array}\right].$$

Find the general solution of this system, and write your final answer in vector form. Show all your work.

6. (16 points) Determine the values of r and s for which the given system of linear equations is consistent (has at least one solution). Show all your work.

$$\begin{aligned} x_1 - 3x_2 + 4x_3 &= -1, \\ 2x_1 - 6x_2 + rx_3 &= s. \end{aligned}$$

7. (20 points) Find the general solution of the following system of linear equations in x_1, x_2, x_3, x_4, and write your final answer in vector form. Show all your work.

$$\begin{aligned} - 2x_2 - 4x_3 + x_4 &= -1, \\ x_1 + 2x_2 + x_3 + &= 1, \\ -2x_1 - 5x_2 - 4x_3 + x_4 &= -3. \end{aligned}$$