Math 129a, paragraph homework 08
Subspaces
Due: Fri Apr 12

1. Let $A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}$.

   (a) Find all $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathcal{R}^3$ such that $A\mathbf{x} = 2\mathbf{x}$. (Hint: Solve a system of linear equations.)

   (b) Now let W be the set of all $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathcal{R}^3$ such that $A\mathbf{x} = 2\mathbf{x}$. Is W a subspace of $\mathcal{R}^3$? Explain why or why not.

2. Suppose W is a subspace of $\mathcal{R}^4$, and suppose also that $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 3 \\ 1 \end{bmatrix}$ are vectors in W.

   (a) Explain why the vector $\begin{bmatrix} 5 \\ 10 \\ 0 \\ 5 \end{bmatrix}$ must be contained in W.

   (b) Explain why the vector $\begin{bmatrix} -3 \\ 4 \\ 6 \\ 3 \end{bmatrix} = 2 \begin{bmatrix} -2 \\ 1 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ must be contained in W.

   (c) Generalize (a) and (b) as much as possible, and explain. In other words, given that $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 3 \\ 1 \end{bmatrix}$ are vectors in W, what is the most general conclusion that you can draw about other vectors that must be contained in W?