Math 129a, paragraph homework 06
Inverses; linear transformations
Due: Mon Mar 11

1. Let $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$, and $\mathbf{z} = \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$.

   (a) Express each of $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ as a linear combination of $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$.

   (b) Let $A = \begin{bmatrix} 1 & -1 & -1 \\ -2 & 5 & 4 \\ -1 & 2 & 2 \end{bmatrix}$. Without doing any further calculation, use your calculcation in part (a) to find a matrix B such that $AB = I_3$. Briefly explain your answer.

2. (a) Suppose $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathcal{R}^5$, and $T : \mathcal{R}^5 \to \mathcal{R}^7$ is a linear transformation. Note that the vectors $T(\mathbf{u})$, $T(\mathbf{v})$, and $T(\mathbf{w})$ are vectors in $\mathcal{R}^7$. Suppose also that $7\mathbf{u} - 2\mathbf{v} + \mathbf{w} = \mathbf{0}$. Is it possible that $\{T(\mathbf{u}), T(\mathbf{v}), T(\mathbf{w})\}$ is linearly independent? Is it possible that $\{T(\mathbf{u}), T(\mathbf{v}), T(\mathbf{w})\}$ is linearly dependent? Briefly explain your answer.

   (b) Suppose $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are vectors in $\mathcal{R}^5$, and $T : \mathcal{R}^5 \to \mathcal{R}^7$ is a linear transformation. Note that the vectors $T(\mathbf{x})$, $T(\mathbf{y})$, and $T(\mathbf{z})$ are vectors in $\mathcal{R}^7$. Suppose also that $\{T(\mathbf{x}), T(\mathbf{y}), T(\mathbf{z})\}$ is linearly independent. Is it possible that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent? Is it possible that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent? Briefly explain your answer.