Math 129a, paragraph homework 03
Which span is bigger?
Due: Mon Feb 11

1. Let $S_1 = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix} \right\}$ and $S_2 = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} \right\}$.

   (a) Is $S_1 \subseteq S_2$? (That is, is every vector in S_1 also a vector in S_2?) Is $S_2 \subseteq S_1$? Explain both answers.

   (b) Generalize your answer to (a) as much as you can. For example, does something similar work for $S_1 = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$ and $S_2 = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for any vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \in \mathcal{R}^n$? What if you vary the number of vectors used to span S_1 and S_2?

2. Let $S = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} \right\}$. How many vectors are there in S? Explain your answer.

3. Let $S_1 = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ and $S_2 = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

   (a) Is $S_1 \subseteq S_2$? Is $S_2 \subseteq S_1$? Explain both answers.

   (b) Can you write down two different finite subsets of $\mathcal{R}^3$ with the same span? How about three, four, five, ... ? Infinitely many? Explain.