Format and topics
Final exam, Math 128B

General information. The final exam will be a timed test of 2 hours, 15 minutes, including up to 15 minutes of upload time. You are allowed

ONE PAGE OF NOTES

and no other aids (books or calculators).

The final will be **cumulative**; in other words, the final will cover the topics on this sheet and the topics on the previous three review sheets. However, the exam will somehwat emphasize the material listed here from the supplemental notes on Galois Theory and Ch. 32, i.e., PS10–11.

As always, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs may help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Definitions. The most important new definitions we have covered are:

Galois	orbit	stabilizer
	conjugate (elements)	conjugacy class
	centralizer	
Ch. 32	automorphism (of a field)	Galois group of E over F
	$\mathrm{Gal}(E/F)$	fixed field
	solvable by radicals over F	solvable (group)

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important new examples we have seen are:

Galois Examples: S_n, A_n, D_n, C_n, V. Subgroup lattices of A_4, D_4, S_3, V. Conjugacy classes of S_3, A_4, S_4, A_5, S_5. Going from conjugacy classes in S_n to conjugacy classes in A_n.

Ch. 32 Subfields of $\mathbf{Q}(\sqrt[4]{2}, i)$ vs. subgroups of D_4. $\mathrm{Gal}(E/\mathbf{Q})$ and subfields of E for $E = \mathbf{Q}(\sqrt{2})$, $\mathbf{Q}(\sqrt[3]{2})$, $\mathbf{Q}(\sqrt[4]{2}, i)$, $\mathbf{Q}(\sqrt{3}, \sqrt{5})$, $\mathbf{Q}(\omega_3, \sqrt[3]{2})$, $\mathbf{Q}(\omega_7)$ ($\omega_n = e^{2\pi i/n}$). $\mathrm{Gal}(\mathbf{Q}(\sqrt[4]{2}, i)/\mathbf{Q}(i))$ and subfields. $\mathrm{Gal}(\mathrm{GF}(p^n)/\mathrm{GF}(p))$ and subfields of $\mathrm{GF}(p^n)$. $x^n - a$ solvable by radicals. Irreducible with two real roots (e.g., $3x^5 - 15x + 5$) not solvable by radicals.

Theorems, results, algorithms. The most important new theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Galois Orbit-Stabilizer Theorem for conjugacy classes. The Cycle-Shape Theorem. The Fundamental Theorem of Galois Theory (expanded statement). Normal subgroups of A_4, A_5, and S_5 (PS11).

Ch. 32 Fundamental Theorem of Galois Theory. Galois group of the splitting field of $x^n - a$ is solvable (Thm. 32.2); factor group of solvable is solvable (Thm. 32.3), N and G/N solvable implies G solvable (Thm. 32.4); solvable by radicals implies solvable group (Thm. 32.5). Insolvability of a particular quntic.

Good luck.