Math 128A, problem set 04
CORRECTED Fri Sep 25
Outline due: Wed Sep 23
Due: Mon Sep 28
Last revision due: Wed Oct 21

Problems to be done, but not turned in: (Ch. 4) 17–75 odd; (Ch. 5) 1–19 odd.
Fun: (Ch. 4) 50, 64, 77.

Problems to be turned in:

1. Justify all answers.
   (a) List all generators of the subgroup $\langle 5 \rangle$ of $\mathbf{Z}$.
   (b) Let $G = \langle a \rangle$ be an infinite cyclic group. List all generators of the subgroup $\langle a^5 \rangle$ of G. (*corrected*)
   (c) $\mathbf{Z}_{60}$ has a subgroup H of order 20. List all generators of that subgroup H.
   (d) Let $G = \langle a \rangle$ be a cyclic subgroup of order 60, and let H be a subgroup of G of order 20. List all generators of that subgroup H.
2. Find the subgroup lattices for $\mathbf{Z}_5$, $\mathbf{Z}_{10}$, $\mathbf{Z}_{70}$, and $\mathbf{Z}_{770}$. Generalize as much as you can.
3. (Ch. 4) 42. Prove your answer.
4. (Ch. 4) 68. Prove your answer.
5. (a) Let G be an abelian group of order 119 such that $x^{119} = e$ for all $x \in G$. Prove that G is cyclic.
   (b) Let G be an abelian group of order 49 such that $x^{49} = e$ for all $x \in G$, and suppose that G is *not* cyclic. What can you say about G?
6. Let $\alpha = (1\ 7\ 4)(2\ 5\ 3\ 9\ 10\ 8\ 6\ 12)$ and $\beta = (1\ 2\ 9\ 3\ 10\ 5)(6\ 8)(7\ 12\ 11)$ be elements of S_{12}.
   (a) Compute $\alpha\beta$, in cycle form.
   (b) Find the orders of α, β, and $\alpha\beta$.
7. The *cycle shape* of $\alpha \in S_n$ is the set (or actually, multiset) of the lengths of the cycles obtained when α is expressed as a product of disjoint cycles (see pp. 102–103).
   (a) Find all possible cycle shapes of elements of S_8, and find the orders of the elements with those cycle shapes.
   (b) Find all possible cycle shapes of elements of A_8.