Math 128A, problem set 03
CORRECTED Wed Sep 09
Outline due: Wed Sep 09
Due: Mon Sep 14
Last revision due: Mon Oct 19

Problems to be done, but not turned in: (Ch. 3) 19–77 odd; (Ch. 4) 1–15 odd.
Fun: (Ch. 3) 78.

Problems to be turned in:

1. (a) Consider $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$. Find the orders of A, B, and C.
   (b) If G is a group, $x, y \in G$, and both x and y have finite order, is it always the case that xy has finite order? Prove or give a counterexample.

2. Find all cyclic subgroups of D_6. Make sure to demonstrate that your answer is complete.

3. Let
$$H = \{A \in GL(3, \mathbf{R}) \mid \det A = 5^n \text{ for some } n \in \mathbf{Z}\}.$$
Prove that H is a subgroup of $GL(3, \mathbf{R})$.

4. Let G be an abelian group, and let
$$S = \left\{a \in G \mid a = x^2 \text{ for some } x \in G\right\}.$$
Prove that S is a subgroup of G.

5. Let G be a group, let a be an element of G, and let
$$N(a) = \left\{g \in G \mid gag^{-1} = a^n \text{ and } g^{-1}ag = a^k \text{ for some } n, k \in \mathbf{Z}\right\}.$$
Prove that $N(a)$ is a subgroup of G. (Suggestion: Note that $(gag^{-1})(gag^{-1}) = ga^2g^{-1}$; you may find a generalization of that to be helpful.)

6. Let G be a group, and let both H and K be subgroups of G. Define
$$HK = \{hk \mid h \in H, k \in K\}.$$
   (a) Give an example to show that HK may not be a subgroup of G.
   (b) Now suppose further that $hk = kh$ for all $h \in H$ and $k \in K$. Prove that HK is a subgroup of G.

7. (a) (Ch. 4) 4.
   (b) (Ch. 4) 6.