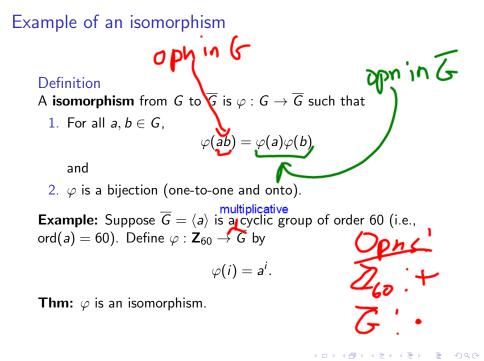
Math 128A, Wed Sep 30

- Use a laptop or desktop with a large screen so you can read these words clearly.
- In general, please turn off your camera and mute yourself.
- Exception: When we do groupwork, please turn both your camera and mic on. (Groupwork will not be recorded.)
- Please always have the chat window open to ask questions.

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- Reading for today and for Mon: Ch. 7.
- PS05 due Mon.
- Problem session Fri Oct 02, 10:00–noon on Zoom.

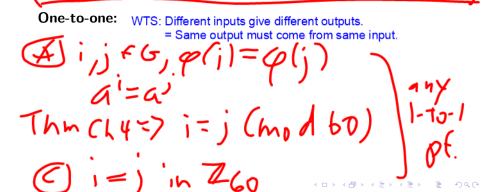
InA Finding cycle shapes of permutations in S_6 and A_6 (abchet (abcde)(f) 5+1 (rbcA)(et)(abch)(e)(t) 10 +1 Remember: Even length cycles are odd perms (-) Odd length cycles are even perms Hot these (G(For 1).5.4.



Well-defined:

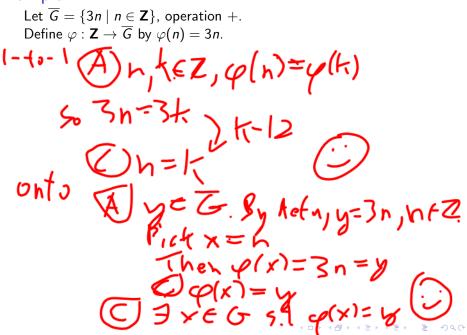
Ambiguity in formula for phi: If the number i is only specified (mod 60), is there only one possible meaning for a^i?

Yes: Thm from Ch.4 that says: If ord(a) = 60, then $a^i = a^j$ if and only if $i=j \pmod{60}$. So in particular, if $i=j \pmod{60}$, then $a^i=a^j$, which means that the RHS of the formula is unambiguous.



WTS: Every element of codomain gets hit as an output. Onto: EG 1 So y= a" for nea A) ictx=n (mod 60+ Then O 5.t. 6/x **Operation-preserving:** equall $\varphi(i+j) =$) * م (i)Ø(i)

Example



Op pros = himom prop Dn.kEZ $\varphi(n+k) = 3(n+k)$ $\varphi(n+q) = 3n+3k^{2}y^{2}$ Note: phi is a homomorphism b/c of distributive law. $() \varphi(n+k) = \varphi(n) + \varphi(k) ()$

Example? ($\bigvee \psi \psi \rangle$) \mathbf{R}^* is nonzero reals, operation \times . Define $\varphi : \mathbf{R}^* \to \mathbf{R}^*$ by $\varphi(x) = 3x$. Is φ an isomorphism?

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Cayley's Theorem

Theorem

Really saying: Every group is a group of symmetries of itself as a geometric object.

Every group G is isomorphic to a permutation group on the set G.

Sketch of proof: Define $T_g: G \to G$ by $f_g(x) = gx.$

Let $\overline{G} = \{T_g \mid g \in G\}$, operation composition. Can show that each T_g is a permutation and that \overline{G} is a group. Now define $\varphi : G \to \overline{G}$ by

$$\varphi(g)=T_g.$$

To prove φ is an isomorphism, we need to:

Gonto

g(1)- 6(9)6

In book: This is the interesting part.

Ze,WTS.

How and why are isomorphic groups the same?

Theorem $\varphi: G \to \overline{G}$ an isomorphism, $a, b \in G$. Then 1. $\varphi(e) = \overline{e}$. 2. $\varphi(a^n) = \varphi(a)^n$. 3. a and b commute $\Leftrightarrow \varphi(a)$ and $\varphi(b)$ commute. 4. $G = \langle a \rangle \Leftrightarrow \overline{G} = \langle \varphi(a) \rangle$. 5. $\operatorname{ord}(a) = \operatorname{ord}(\varphi(a)).$ 6. $x^k = b$ and $\overline{x}^k = \varphi(b)$ have the same number of solutions. 7. $\varphi^{-1}: \overline{G} \to G$ is also an isomorphism. 8. G and \overline{G} have same number of elements of each order. 9. G abelian $\Leftrightarrow \overline{G}$ abelian. 10. φ sends subgroups of G to subgroups of \overline{G} , and vice versa.

11. φ sends the center of G to the center of G.

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Proving that groups are **not** isomorphic

Not enough to pick some $\varphi: G \to \overline{G}$ and show φ isn't an isomorphism — maybe there's a different map that is! But just as two people with different eye colors can't be genetic twins, two groups with different characteristics can't be isomorphic. **Example:** Two groups of order 10 that aren't isomorphic?

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Example: Prove that D_6 and A_4 aren't isomorphic.

Automorphisms

Definition

An **automorphism** of G is an isomorphism from G to itself.

An automorphism of G isn't used to show that G is the same as itself; it shows a certain symmetry in the structure of G.

Definition

 ${\it G}$ a group, ${\it a} \in {\it G}.$ Define $\varphi_{\it a}: {\it G} \rightarrow {\it G}$ by

$$\varphi_a(x) = axa^{-1}$$

for all $x \in G$. We call φ_a an inner automorphism of G.

Try at home: Prove that φ_a is an automorphism of *G*. Can show that the following are groups:

$$\begin{aligned} \mathsf{Aut}(G) &= \{ \mathsf{all automorphisms of } G \} \\ \mathsf{Inn}(G) &= \{ \mathsf{all inner automorphms of } G \} \\ &= \{ \varphi_a \mid a \in G \} . \end{aligned}$$

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Cosets

Definition G a group, H a subgroup, $a \in G$. Define

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We call Ha the **left coset of** H **in** G **containing** a, and we call aH the **right coset of** H **in** G **containing** a.

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Examples

 $G = U(24), H = \{1, 5, 7, 11\}.$

$$G = S_4$$
, $H = \langle (1 \ 2 \ 3) \rangle = \{\epsilon, (1 \ 2 \ 3), (1 \ 3 \ 2) \}.$

Cosets via equivalence relations

 $H \leq G$, $a, b, c \in G$.

Definition

Define $a \sim b$ to mean that $a^{-1}b \in H$.

Theorem

 \sim is an equivalence relation on G.



Cosets are equivalence classes

What are equivalence classes of \sim ? The class of $a \in G$ is:

$$\{b \in G \mid a \sim b\} = \{b \in G \mid a^{-1}b \in H\}$$
$$= \{b \in G \mid b \in aH\}$$
$$= aH.$$

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So left cosets of H are equivalence classes of an equivalence relation, which means that left cosets of H partition G: