### Math 128A, Wed Sep 09

- Use a laptop or desktop with a large screen so you can read these words clearly.
- In general, please turn off your camera and mute yourself.
- Exception: When we do groupwork, please turn both your camera and mic on. (Groupwork will not be recorded.)
- Please always have the chat window open to ask questions.
- Reading for Mon: Ch. 4.
- Outline for PS03 due 11pm, complete PS03 due Mon Sep 14.
- Next problem session Fri Sep 11, 10:00-noon on Zoom.
- Zoom proctoring rehearsal Mon Sep 14. Details over the weekend, but have blank paper ready and be ready to turn on your camera on Mon.
- Exam 1 moved to Mon Sep 21, to cover Chs. 1–4 and PS01–03.

### Error in PS03

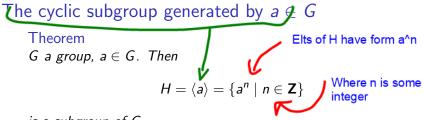
In Problem 5, definition of N(a) should be:  $N(a) = \left\{ g \in G \mid gag^{-1} = a^n \text{ and } g^{-1}ag = a^k \text{ for some } n, k \in \mathbb{Z} \right\}.$ Corrected version now up on website Other questions?

Suppose *H* has a definition of the form {foo | bar}. To apply the Two-Step Subgroup Test:

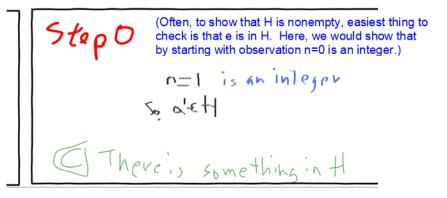
 Write out steps 0, 1, 2 as if-then statements and set up Assumptions and Conclusions.

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- Rewrite **A** and **C** using {foo | bar} definition of *H*.
- Fill in the middle.



is a subgroup of G.



Stepl Assume: of, ad & H < d & Z (ac)(ad) - acto BC CHAEZ Conclusion: EH

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2: A: × <H }H= So x= d } { { } } { } ]  $x^{-1} = \alpha^{-n}$ 50, 516 - m (Z с. ҂҄Ҷ

## The centralizer of $a \in G$

#### Theorem

G a group,  $a \in G$ . Then (centralizer = everything that commutes with a)

$$C(a) = \{g \in G \mid ga = ag\}$$

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is a subgroup of G. O. Consider eeG. eq= Re (identity) So e E (a) (( ( ( ) ≠ Ø.

 $A \times y \in (a) \quad (a) = \{g \in G \mid ga = 1\}$ So  $x \in G, x \in A = a \times a$  $y \in G, ya = a \neq x$  $5_{0} = \chi_{ay} = \alpha_{y}$  mult by y on right xya = Axy by A 5. xy=G, (xy)a=a(xy) J xy=C(a) (< ) ▶ ▲ 臣 ▶ ▲ 臣 ▶ ○ 臣 → の Q (?)

C121 ×9  $A \times C(a)$ 5. XEG, XA=AX mult by x^{-1} on left  $X' \times K = x^{-1} A X$  $q = \chi^{-1} a \chi$  mult by x^{-1} on right AX-I=Y-GXX-I= 50 x 60, x 6- $() \times [\ell(a)]$ 

How can we understand a class of groups completely?

To understand a class of groups completely, we must be able to:

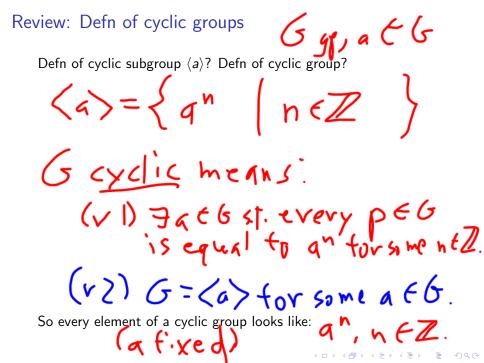
- List all elements of that class of groups.
- For G in that class, write down elements of G, compute the product of two elements, and understand the order of a given element of G.

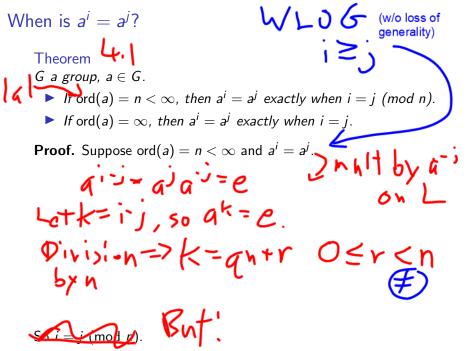
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For G in that class, list all subgroups of G.

a' a' - a'+j

Goal of Ch. 4 is to understand cyclic groups completely.





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 $e = a^{h} = a^{lh}a^{r}$ (a <sup>n</sup> = B  $= (a^{n})^{f} a^{r}$  $= a^{\vee}$ So are, and Ofran h is smallest ors int stan=e, su r cant be >0. s1 r=0. 50 k= (n=> k= U (m) dn) => i-j = 0 (mod n)

Corollaries to  $a^i = a^j$  theorem

$$G$$
 a group,  $a, b \in G$ ,  $n = \operatorname{ord}(a)$ .

### Corollary

 $a^k = e$  if and only if n divides k.

### Corollary

 $|\langle a \rangle| = \operatorname{ord}(a).$ 

Second Corollary shows that  $\langle a \rangle$  is "the same as" the cyclic group  $\mathbf{Z}_n$ , i.e., all cyclic groups of a given order are "the same". (But we first have to define what it means to be "the same"....)

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What is  $\langle a^k \rangle$  like?

Theorem

G a group,  $a \in G$ ,  $\operatorname{ord}(a) = n < \infty$ . Then

$$\left\langle a^{k} \right\rangle = \left\langle a^{\gcd(n,k)} \right\rangle, \quad \operatorname{ord}(a^{k}) = \frac{n}{\gcd(n,k)}.$$

**Proof.** Suppose  $\operatorname{ord}(a) = n < \infty$ , k > 0,  $d = \operatorname{gcd}(n, k)$ , n = qd.

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So 
$$\langle a^k \rangle \subseteq \langle a^d \rangle$$
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# So $\langle a^d \rangle \subseteq \langle a^k \rangle$ .

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Corollaries to  $\langle a^k \rangle$  theorem

*G* a group, 
$$a, b \in G$$
,  $\operatorname{ord}(a) = n < \infty$ .  
Corollary  
 $\langle a^k \rangle = \langle a^j \rangle$  if and only if  $\operatorname{gcd}(n, k) = \operatorname{gcd}(n, j)$ .  
Corollary

 $a^k$  generates  $\langle a \rangle$  if and only if gcd(n, k) = 1.