Math 128A, Mon Aug 24

- Use a laptop or desktop with a large screen so you can read these words clearly.
- In general, please turn off your camera and mute yourself.
- Exception: When we do groupwork, please turn both your camera and mic on. (Groupwork will not be recorded.)
- Please always have the chat window open to ask questions.
- Reading for today: Ch. 1. Reading for Wed: Ch. 2.
- PS00, PS01 outline due tonight, 11pm; PS01 due Wed Aug 26.
- Next problem session Fri Aug 28, 10:00–noon on Zoom.

The symmetries of a regular n-gon

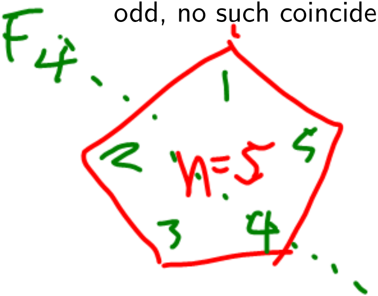
The symmetries of a regular n-gon are all of the different ways you can pick it up and put it back down ("rigid motions") and have it still look the same.

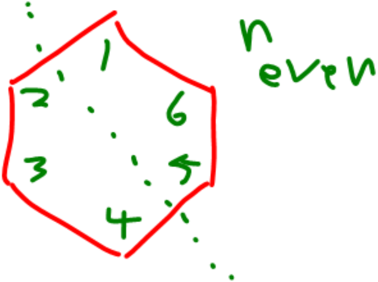
"Different" means that two motions are equal if and only if they have the same net result, e.g., a 360° turn is the same as not moving at all.

The set (or in Ch. 2, the **group**) of all such symmetries is called D_n, the dihedral group of **order** $2n$. "Order" here refers to the total number of symmetries.

Notation for symmetries of (any) n-gon

- R_d is a counterclockwise rotation through d degrees. (So a rotation of the n-gon through $\frac{360}{n}$ degrees moves its vertices one "notch".) $n=6$ ← R_{60}
- F_v is the reflection through the **original** location of vertex v. (Note that for the hexagon $n = 6$, $F_1 = F_4$; but when n is odd, no such coincidences happen.)

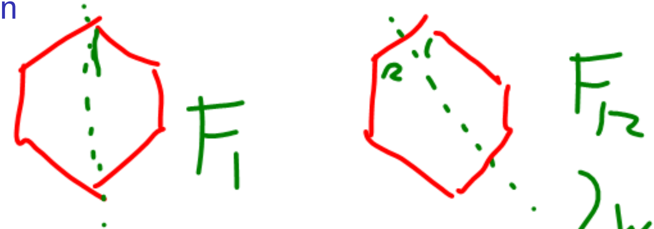




- F_{vw} is the reflection through the **original location** of the midpoint of the edge with vertices v and w. (Note that when n is odd, there is no need for elements of this form.)

Composition

Ex D_6

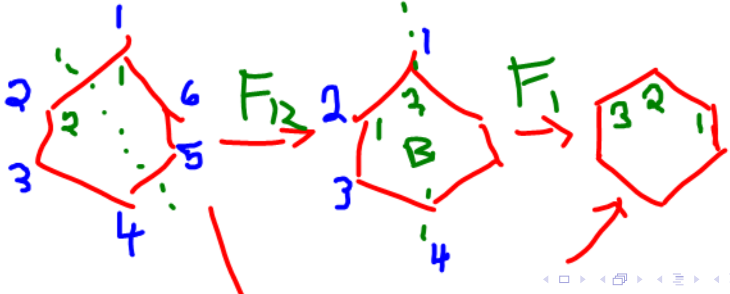


what is $F_1 F_{12}$? first

Because rigid motions are functions, fg is a composition of functions

$$(f \circ g)(x) = f(g(x)),$$

which means that you apply g first, then f.



R_{300}

$$\boxed{F_1 F_{12} = R_{300}}$$

The Cayley table of D_6

Let's fill in the multiplication table, or **Cayley table**, for D_6.

(To the limnu board!)

PS01: Bits of the Cayley table of D_5

Note that for D_5:

- There are rotations R_d, where d is a multiple of $\frac{360}{5} = 72$: R_0, R_{72}, R_{144}, R_{216}, R_{288}.
- There are reflections F_v for $v = 1, 2, 3, 4, 5$. (Again, when n is odd, each edge midpoint reflection is equal to a vertex reflection.)