Sample Exam 1
Math 127, Spring 2021

1. (12 points) Let F be a field, and let $d(x), a(x), b(x)$ be polynomials in $F[x]$.

(a) Define what it means for $d(x)$ to divide $a(x)$.

(b) Define what it means for $d(x)$ to be a common divisor of $a(x)$ and $b(x)$, and what it means for $d(x)$ be a greatest common divisor of $a(x)$ and $b(x)$.

2. (12 points) Recall that to say that $a \in \mathbf{Z}/(13)$ is a quadratic residue means that $a \neq 0$ and $x^2 = a$ has a solution $x \in \mathbf{Z}/(13)$. List all of the quadratic residues in $\mathbf{Z}/(13)$. Show all your work.

For questions 3–4, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (12 points) Let F be a field, and let $f(x), g(x) \in F[x]$ be nonzero polynomials. Then it must be the case that $\deg(f(x)g(x)) = \deg(f(x)) + \deg(g(x))$.

4. (12 points) If $n > 1$ is an integer, $a, b \in \mathbf{Z}/(n)$, and $a \neq 0$ in $\mathbf{Z}/(n)$, then the equation $ax = b$ always has a solution $x \in \mathbf{Z}/(n)$.

5. (13 points) Let $a = 147$, $b = 120$, and $d = \gcd(a, b)$. Find d and find $x, y \in \mathbf{Z}$ such that $ax + by = d$. Show all your work.

6. (13 points) Use the Euclidean Algorithm to find $x \in \mathbf{Z}/(41)$ such that $22x = 1$ in $\mathbf{Z}/(41)$. Show all your work.

7. (13 points) Consider the polynomials $f(x) = x^4 - 2x^3 - 2x^2 - x - 1$ and $g(x) = x^3 - 2x + 1$ in $\mathbf{F}_5[x]$. Find $\gcd(f(x), g(x))$ in $\mathbf{F}_5[x]$. Show all your work.

8. (13 points) In the song "The n Days of Christmas", on day 1, the singer gets 1 gift of type 1; on day 2, the singer gets two gifts of type 2 and one give of type 1; and so on.

(a) Give a big O estimate of the *number* of gifts the singer receives on day n.

(b) Assuming the entire song runs from day 1 through day n, give a big O estimate of the *total number* of gifts the singer receives over the course of the entire song.