Sample Exam 1
Math 126, Spring 2015

1. (12 points) Let c, d, n be integers, $n \geq 1$. Define what it means for c be congruent to d (mod n), i.e., $c \equiv d \pmod{n}$.

2. (10 points) Find positive integers a, b, c such that $a^2 + b^2 = c^2$, $\gcd(a, b) = \gcd(a, c) = \gcd(b, c) = 1$, $a = 3k$ for some odd integer k, b is even, and $c > 1000$. Briefly **EXPLAIN** how you know that your choice of a, b, c satisfies the given conditions.

3. (8 points) Find $\gcd(798, 111)$, using methods from our class. Show all your work.

4. (10 points) Suppose that $a > b > c > d$ are positive integers such that

$$\begin{aligned} a &= 2b + c, \\ b &= 3c + d, \\ c &= d + 7, \end{aligned}$$

and 7 divides d. Find $g = \gcd(a, b)$, and find integers x and y such that $ax + by = g$. Show all your work, and briefly **JUSTIFY** your answer. (I.e., how do you know that the value of $\gcd(a, b)$ is what you say it is?)

For questions 5–8, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

5. (12 points) For any positive integers a, b, c, we can always find integers $x, y \in \mathbb{Z}$ such that $ax + by = c$.

6. (12 points) If a, b, n are positive integers, and n divides ab, then it must be the case that either n divides a or n divides b.

7. (12 points) It is possible that there are infinitely many primes of the form $N^2 - 3N + 2$, where N is a positive integer.

8. (12 points) If s is a positive integer, and $s = 2^k m = 2^\ell n$, where k, ℓ are positive integers and m, n are odd positive integers, then it must be the case that $k = \ell$ and $m = n$.

9. (12 points) **PROOF QUESTION.** Suppose a, b, c are positive integers such that

$$a^3 + b^3 = c^2,$$

and suppose p is a prime. Prove that if p divides a and p divides b, then p divides c.

Note: If you rely on certain facts about factorization or divisibility (e.g., the Fundamental Theorem of Arithmetic), please clearly state how you rely on those facts.