SA25
Math 112, Spring 2006

1. (8.4) 4(b).

2. (8.4) 8.

3. If the vector field $\mathbf{E}(x, y, z)$ is the electric field induced by a collection of stationary charges, in dynes per electrostatic unit, and $\delta(x, y, z)$ is the density of charge at the point (x, y, z), in electrostatic units per cm^3, then the differential form of Gauss' Law (in CGS units) states that

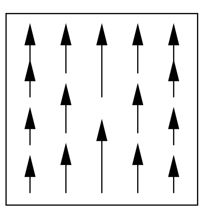
$$\operatorname{div} \mathbf{E}(x, y, z) = 4\pi\delta(x, y, z),$$

and the integral form of Gauss' Law (in CGS units) states that if a closed surface S encloses a total charge q, then

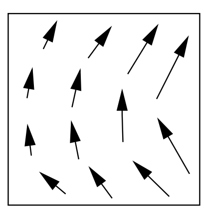
$$\iint_S \mathbf{E} \cdot d\mathbf{S} = 4\pi q.$$

Explain why the differential form of Gauss' Law implies the integral form of Gauss' Law.

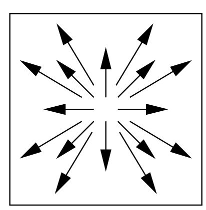
4. Below are sketches of four vector fields $\mathbf{F}_i$ $(1 \leq i \leq 4)$. For three of those vector fields, $\operatorname{div} \mathbf{F}_i = 0$ everywhere, and for two of those vector fields, $\operatorname{curl} \mathbf{F}_i = \mathbf{0}$ everywhere. Use Stokes' Theorem and the Divergence Theorem to identify which vector fields seem most likely to be curl-free and divergence-free, and briefly **justify** your answer.



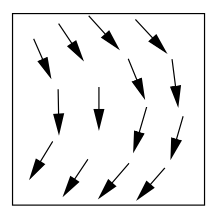
$\mathbf{F}_1$



$\mathbf{F}_2$



$\mathbf{F}_3$



$\mathbf{F}_4$