

**SA16**  
**Math 112, Spring 2006**

You may find the following definite integrals useful. (I.e., these are given.)

$$\begin{aligned}\int_0^{2\pi} \sin^2 x \, dx &= \int_0^{2\pi} \cos^2 x \, dx = \pi, \\ \int_0^{\pi/2} \sin^2 x \, dx &= \int_{\pi/2}^{\pi} \sin^2 x \, dx = \int_{\pi}^{3\pi/2} \sin^2 x \, dx = \int_{3\pi/2}^{2\pi} \sin^2 x \, dx = \frac{\pi}{4}, \\ \int_0^{\pi/2} \cos^2 x \, dx &= \int_{\pi/2}^{\pi} \cos^2 x \, dx = \int_{\pi}^{3\pi/2} \cos^2 x \, dx = \int_{3\pi/2}^{2\pi} \cos^2 x \, dx = \frac{\pi}{4}.\end{aligned}$$

1. (6.2) 4(a).
2. (6.2) 16.
3. Let  $D$  be the region in  $\mathbb{R}^2$  given by  $x^2 + y^2 \leq 4$ ,  $x \geq 0$ ,  $y \geq 0$ . Draw  $D$ , and calculate

$$\iint_D (x^2 + xy) \, dx \, dy$$

using polar coordinates.

4. Let  $E$  be the cylinder of radius 3 with center the  $z$  axis and  $0 \leq z \leq 4$ . Draw  $E$ , and calculate

$$\iiint_E (\sin z)(x + y + 2) \, dx \, dy \, dz$$

using cylindrical coordinates.

5. Let  $E$  be the upper hemisphere of radius 5 centered at the origin. Draw  $E$ , and calculate

$$\iiint_E z(x^2 + y^2) \, dx \, dy \, dz$$

using spherical coordinates.