Topics for Exam 2
Math 112, Spring 2006

General information. Exam 2 will be a timed test of 50 minutes, covering 2.2–2.6, 3.1, and 3.3–3.4 of the text. Most of the exam will be based on the homework assigned for those sections. If you can do all of that homework, and you know and understand all of the ideas behind it, you should be in good shape.

You are allowed to use a calculator and notes on **ONE** 3×5 note card (both sides).

As mentioned above, your first priority should be to understand the homework and quizzes and the ideas behind them. Besides the list of things you should know, below, you should also be familiar with everything specially emphasized in the text. If time permits, try to do some of the problems that have answers in the back of the book.

Review. Matrix multiplication.

Section 2.2. Open sets: definition, picture, examples. Boundary of a set. Definition of limit (version in terms of $\mathbf{x} \to \mathbf{a}$ implies $f(\mathbf{x}) \to \mathbf{b}$, etc.). Definition of continuity (in terms of limit).

Section 2.3. Partial derivatives: idea, definition, computation. Linear/tangent plane approximation to $f(x, y)$; finding equation of the tangent plane. Idea of definition of differentiability ("zooming in", relative error approaches 0). Definition of **the derivative** of $f : \mathbb{R}^k \to \mathbb{R}^n$. Gradients. Thms: Differentiability implies continuity; continuous partials implies differentiable implies partial exist, but not converse.

Notes on limits and continuity, notes on differentiability, paragraph HW 2 and 3. There will definitely be a conceptual question on the definitions of differentiability and continuity, and how it is not enough to look at one variable at time. (E.g.: Functions f such that $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$, but f is not differentiable at $(0,0)$.) If you understand the examples in paragraph HW 2 and 3 and the handouts, as well as the ideas behind them, that should be fine.

Section 2.4. Definitions: path, curve, the difference between them. Velocity of a path, tangent vector to a path. Tangent line to a path.

Section 2.5. Properties of the derivative. Chain Rule. Special cases of Chain Rule.

Section 2.6. Definition of gradient. Definition of directional derivative; calculating directional derivatives (p. 165). Interpretation of gradient: direction of greatest increase, magnitude gives largest possible rate of change, normal to level surfaces. Finding the tangent plane to a level surface.

Section 3.1. Definition of higher partials; continuous mixed partials equal.

Section 3.3. Defns of max/min: local and absolute. Definition of critical point; first derivative test. Second derivative test (p. 216), classification of critical points. Existence of global max/min on closed and bounded domains. Finding absolute min/max.

Section 3.4. Idea of constrained optimization. Method of Lagrange multipliers; examples. Global min/max. Applications.

Not on exam. (2.2) Definition of $\mathbf{x} \to \mathbf{a}$, $f(\mathbf{x}) \to \mathbf{b}$, etc.; algebraic properties of limits and continuous functions; ϵ-δ stuff (pp. 121–125). (2.5) Multivariable product and quotient rules. (2.6) Gradient vector fields (these will be on a future exam). (3.1) Partial differential equations. (3.3) Extrema of functions of more than two variables (pp. 215–219). (3.4) Multiple constraints; second derivative test for constrained extrema.