Paragraph HW 07
Which vector fields are gradients?
Math 112, Spring 2006

As in the previous homework, you can draw vector fields by hand, if you prefer not to use Maple, but Maple will make your task much easier.

Consider the following vector fields:

$$\mathbf{F}_1 = x^2\mathbf{j},$$
$$\mathbf{F}_2 = -\frac{y}{x^2+y^2}\,\mathbf{i} + \frac{x}{x^2+y^2}\,\mathbf{j},$$
$$\mathbf{F}_3 = 2xy\mathbf{i} + x^2\mathbf{j}.$$

1. For each $\mathbf{F}_i$, $i = 1, 2, 3$, do each of the following:

   (a) Calculate $\operatorname{curl}\mathbf{F}_i$.

   (b) If possible, find a simple closed curve C such that

   $$\int_C \mathbf{F}_i \cdot d\mathbf{s} > 0.$$

   If this is not possible, note that it does not seem to be possible (you do not need to prove that it is not possible). If you do this by picture instead of by explicit calculation, explain your answer.

   (c) If possible, find non-closed curves C_1 and C_2 such that C_1 and C_2 have the same starting point and the same end point, but

   $$\int_{C_1} \mathbf{F}_i \cdot d\mathbf{s} \neq \int_{C_2} \mathbf{F}_i \cdot d\mathbf{s}$$

   If this is not possible, **explain** why not, based on your previous answers.

2. Below are three logical statements, (a), (b), and (c), about a hypothetical vector field $\mathbf{F}$. Based on your work above, try to deduce as much as possible as to which of these statements implies which of the others. (Does (a) imply (b)? Does (b) imply (a)? Does (a) imply (c)? And so on.)

   (a) At every point in the domain of $\operatorname{curl}\mathbf{F}$, $\operatorname{curl}\mathbf{F} = \mathbf{0}$ (the zero vector).

   (b) For every simple closed curve C, $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} = 0$.

   (c) Whenever curves C_1 and C_2 the same starting point and the same end point, we have that $\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = \int_{C_2} \mathbf{F} \cdot d\mathbf{s}$

Code for this assignment. As usual, you can get a copy of the worksheet for this assignment either from the handouts folder, if you're working in the Math Lab, or from the course website:

`http://www.math.sjsu.edu/~hsu/courses/112/Math112-p07.mw`