Paragraph HW 02
Open sets, limits, continuity/Even more Maple
Math 112, Spring 2006

1. (2.2) 4. "Show that" means "Explain why." In particular, you should refer to the definition of open set in your explanation.

2. Let

$$f(x,y) = \begin{cases} 0 & \text{if } x = y = 0; \\ \dfrac{xy}{x^2+y^2} & \text{otherwise.} \end{cases}$$

   (See the instructions below for a way to implement this function in Maple.)

   (a) Let $g(x) = f(x,0)$ and $h(y) = f(0,y)$. Explain why $g(x)$ and $h(y)$ are both continuous functions of one variable. You don't have to give a formal proof, but you should explain why this is true.

   (b) Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Is $f(x,y)$ continuous at $(x,y) = (0,0)$? Give a definite answer, yes or no, and justify your answer. You may find the following ideas useful in justifying your answer:

   - The definition of $\lim_{(x,y)\to(0,0)} f(x,y)$.
   - Graphing $f(x,y)$ using Maple (see below; code for this will be given in a Maple worksheet).
   - Considering different ways to make (x,y) approach $(0,0)$, and looking at the corresponding values of $f(x,y)$.
   - Making a table of values of $f(x,y)$, for (x,y) near $(0,0)$ (see below; code for this will be given in a Maple worksheet)).
   - Polar coordinates.

Even more Maple

Code for this assignment. You can get a copy of the worksheet for this assignment either from the handouts folder, if you're working in the Math Lab, or, as always, from the course website:

`http://www.math.sjsu.edu/~hsu/courses/112/Math112-p02.mw`

Functions with multi-part definitions. In the homework, you have to work with the function

$$f(x,y) = \begin{cases} 0 & \text{if } x = y = 0; \\ \dfrac{xy}{x^2+y^2} & \text{otherwise.} \end{cases}$$

Since Maple is a full programming language, it turns out that this function can be implemented without too much difficulty. One way of doing this is (over):

```
> f := (x,y) ->
> if (x^2 + y^2=0)
>    then 0
>    else x*y/(x^2+y^2)
> fi:
```

Note that if you enter this line by line, you will receive warnings saying something about "premature end of input." You may safely ignore these as long as you terminate the input correctly with `fi:`. On the other hand, if the warnings really bother you, you can just hit SHIFT-ENTER instead of ENTER at the end of each line except the last one, or you can just type the entire definition on one line.

Graphs of functions of 2 variables in 3-space. Remember that to plot, for instance, $z = x^2 + y^2$, you use the command:

```
> plot3d(x^2+y^2,x=-2..2,y=-2..2,axes=normal);
```

Unlike `plot`, `plot3d` requires you to specifically request axes to appear; if you omit the `axes=normal` part of the command, none will appear. You can also change the type of axes of your plot by using the "Axes" pulldown menu; for instance, you may want to try "framed" axes.

Contour plots of functions of 2 variables. Remember that to do a contour plot of, for instance, $f(x, y) = x^2 + y^2$, you use the command:

```
> contourplot(x^2+y^2,x=-2..2,y=-2..2);
```

This will plot level sets $x^2 + y^2 = C$ for certain carefully selected values of C in the region $-2 \le x \le 2$, $-2 \le y \le 2$.

Tables of values of a function. The worksheet for PS02 includes a short program for making a table of values of a function. To illustrate by example, the command

```
> deltatable(f,1.0,2.0,0.01,0.03);
```

will produce a table of values of $f(x, y)$ for $x = 0.99$, 1.0, and 1.01, and $y = 1.97$, 2.0, and 2.03 (i.e., a total of 9 values examined). To look at a different set of values, say for $x = 0.0 \pm 0.01$ and $y = 0.0 \pm 0.01$, you can run `deltatable` using different numbers:

```
> deltatable(f,0.0,0.0,0.01,0.01);
```

One peculiarity of Maple: When working with decimal approximations, which in this class we do most of the time, always enter all numbers, even whole numbers, with a decimal point included. In other words, always use `3.0` instead of `3`.