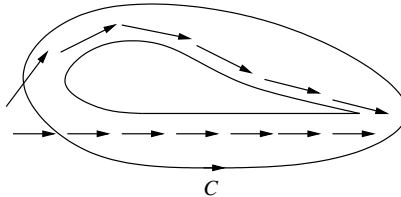# Why the Aerobie has a hole in it
(or, why planes fly)
Math 112

As shown in the picture below, the basic idea of why a plane flies is that air moves faster on top of the wing than below. The air pressure is therefore lower on top of the wing, causing the air to push the wing up, a force called *lift*.

This idea can given a precise mathematical formulation with a few assumptions: Namely, we assume that the airflow $\vec{\mathbf{F}}(x, y, z)$ is incompressible (div $\vec{\mathbf{F}} = \vec{0}$), which (as it turns out) implies that $\vec{\mathbf{F}}(x, y, z)$ is *irrotational*, i.e., curl $\vec{\mathbf{F}} = \vec{0}$. Given those assumptions, it can be shown that, as a result of Newton's laws:

Theorem (Joukowski). *The lift L (per unit of wingspan) on a wing is equal to $-\rho v \Gamma$, where ρ is a positive constant, v is the airspeed, and $\Gamma = \int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ for any simple closed path C enclosing the wing, as shown above.*

(Note that Stokes' Theorem implies that the value of Γ is independent of the C we choose; see paragraph HW 09.)

In any case, from Joukowski's Theorem, we see that it is necessary to have closed paths C with $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} \neq 0$. However, since curl $\vec{\mathbf{F}} = \vec{0}$, if the domain of $\vec{\mathbf{F}}$ is simply connected, we have $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = 0$ for *every* closed C. Therefore, to get lift on the wing, the complement of the "wing" must be non-simply connected. That's why the Aerobie has a hole in it.

On the other hand. You might note at this point (possibly with alarm) that the complement of a typical airplane *is* simply connected. So how can a plane fly? The validity of our idealized assumptions aside, the main way around this (even in real life!) is that the airflow $\vec{\mathbf{F}}$ cannot be continuous everywhere in the complement of the airplane. In particular, one may observe experimentally that an airplane leaves a double vapor trail behind it, along which $\vec{\mathbf{F}}$ is not defined continuously (see below). (Alternately, we might think of this double vapor trail as a region where $\vec{\mathbf{F}}$ is no longer irrotational; in other words, the double vapor trail is a small region of concentrated "vorticity," or turbulence.)