Format and topics
Final exam, Math 108

General information. The final exam will be a timed test of 2 hours and 15 minutes, covering everything we have done this semester, including Chapters 1–12 and 14–23 of the Yellow and Blue Book, the various course notes handed out throughout the semester, and all of the proof notes. (Exception: Material on Tracy and Livermore/knights and knaves will not be on the final.) No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. Note that the material on the final exam will include all of the material listed here, plus all of the material on the previous three review sheets, though there will be some emphasis on the newest material.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Types of questions. The final exam will feature the usual types of questions.

Definitions. The most important definitions we have covered recently in the Yellow and Blue Book are:

Ch. 20	converges	limit
	diverges	
Ch. 21	equivalence (of sets)	same cardinality
	equipotent	equinumerous
	finite	infinite
	restriction (of a function)	
Ch. 22	cardinality (of a finite set	
Ch. 23	countably infinite	countable
	uncountable	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Ch. 20 Examples 20.2, 20.3; bounded does not imply convergent.
Ch. 21 Strange examples of sets with the same cardinality (Thm. 21.5, etc.).
Ch. 22 Examples of provably infinite sets (**N**, **Z**, etc.)
Ch. 23 Examples of surpising countable sets (finite union of countable sets, countable union of countable sets, product of countable sets); the uncountable set **R**.

You should also be familiar with all of the examples from the Exercises from Ch. 20–23, and you should be familiar with the examples from PS09–11.

Theorems, results, algorithms, axioms. The most important theorems, results, algorithms, and axioms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Ch. 20 Limits unique (Thm. 20.7); Convergent $\Rightarrow$ bounded (Thm. 20.8); Limit laws (Thm. 20.9).
Ch. 21 Equivalence/"same cardinality" is an equivalence relation (Thm. 21.1); subset of finite is finite (Cor. 21.10); finite union of finite is finite (Thm. 21.11, Ex. 21.12); finite product of finite is finite (Cor. 21.14).
Ch. 22 The pigeonhole principle (Thm. 22.2); size of a finite set is well-defined (Thm. 22.6).
Ch. 23 Every subset of a countable set is countable (Cor. 23.4); A countable if and only if there exists one-to-one $f : A \to \mathbf{N}$ (Ex. 23.5); finite union of countable is countable (Cor. 23.7); finite

product of countable is countable (Cor. 23.10); countable union of countable is countable (Thm. 23.13). Rational numbers $\mathbf{Q}$ are countable (Thm. 23.11). Real numbers $\mathbf{R}$ are uncountable (Thm. 23.12).

Cardinality "Model set" technique (e.g., using $\mathbf{N}$ to prove theorems about arbitrary countably infinite sets). See also summary of cardinality.

Not on final. People from Tracy and Livermore/knights and knaves.

Other. Please be familiar with the "techniques of proof" in the proof notes.

Good luck.