Math 108, problem set 11
Outline due: Wed May 11
Completed version due: Mon May 16
Last revision due: TBA

Exercises (to be done but not turned in): 22.4, 22.5, 23.1, 23.3, 23.5, 23.9.

Problems to be turned in: All numbers refer to problems in the Yellow and Blue Book.

1. Carefully prove that the open interval $(0, 1)$ is infinite.
2. 23.7.
3. 23.9.
4. 23.11.
5. For each of the following infinite sets, determine whether the set is countably infinite or uncountable, and prove each assertion you make, using the results of Chapter 21–23. (Note that you should not need to construct bijections or duplicate the Cantor diagonalization proof; just apply the results proved there.)

   (a) $A = \big\{\text{All straight lines in } \mathbf{R}^2 \text{ through the point } (0,3)\big\}$.

   (b) $B = \mathbf{R} \backslash \mathbf{Q}$.

   (c) $C = \big\{(x, y) \in \mathbf{Q}^2 \mid x^2 + y^2 = 1\big\}$.

   (d) $D = \{(x, y, z) \mid x \in \mathbf{N},\ y \in \mathbf{Z},\ z \in \mathbf{Q}\}$.

6. Let $S = \{f : \mathbf{N} \to \{0, 1\}\}$, i.e., let S be the set of all functions with domain $\mathbf{N}$ and codomain $\{0, 1\}$. Use the Cantor diagonalization argument to prove that S is uncountable.

   Suggestions: Given $f : \mathbf{N} \to S$, think of f as a sequence in S, i.e., think of (f_n) as a sequence of functions with domain $\mathbf{N}$ and codomain $\{0, 1\}$, so that for each $n \in \mathbf{N}$, $f_n : \mathbf{N} \to \{0, 1\}$. Replace the nth digit of the nth real number with the value of $f_n(n)$.