

Math 108, problem set 10
Outline due: Wed Apr 27
Completed version due: Mon May 09
Last revision due: TBA

Exercises (to be done but not turned in): 21.4, 21.7, 21.12.

Problems to be turned in: All numbers refer to problems in the Yellow Book.

1. 21.5.
2. (a) Prove that $(-1, 1) \approx \mathbf{R}$. (Suggestion: See old homework.)
(b) Prove that for $a, b \in \mathbf{R}$, $a < b$, we have $(a, b) \approx (-1, 1)$.
(c) Prove that for $a, b \in \mathbf{R}$, $a < b$, we have $(a, b) \approx \mathbf{R}$.
3. 21.8.
4. 21.10.
5. 21.16.
6. Define a *text* in English to be a finite sequence of letters A–Z, the character SPACE, and the digits 0–9.
 - (a) Carefully prove that the number of texts of length at most 10^{60} is finite. (Note that 10^{60} is larger than the number of particles in the known universe, which means that effectively, there are only finitely many possible books in the universe.)
 - (b) Now let T be the set of all possible texts of any length. Prove that there exists an injective map from \mathbf{N} to T , and prove that T is infinite.