

Math 108, problem set 07
Outline due: Wed Apr 06
Completed version due: Mon Apr 11
Last revision due: Wed Apr 27

Exercises (to be done but not turned in): 16.3, 16.5, 16.9, 17.1, 17.6.

Problems to be turned in: All numbers refer to problems in the Yellow and Blue Book.

1. Let $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ be defined by $f(x, y) = xy$.
 - (a) Find a function $g : \mathbf{R} \rightarrow \mathbf{R}^2$ such that $f \circ g = \text{id}_{\mathbf{R}}$.
 - (b) Prove that there does not exist a function $h : \mathbf{R} \rightarrow \mathbf{R}^2$ such that $h \circ f = \text{id}_{\mathbf{R}^2}$.
2. Let $f : \mathbf{N} \rightarrow \mathbf{N}$ and $g : \mathbf{N} \rightarrow \mathbf{N}$ be defined by

$$f(n) = n + 1,$$
$$g(n) = \begin{cases} n - 1 & \text{if } n > 0, \\ 0 & \text{if } n = 0. \end{cases}$$

Is $f \circ g = \text{id}_{\mathbf{N}}$? Is $g \circ f = \text{id}_{\mathbf{N}}$? Are f and g inverses? Explain completely, and prove any assertions you make.

3. Suppose $f : A \rightarrow B$ and $g_1, g_2 : B \rightarrow A$ are functions.
 - (a) Suppose $f \circ g_1 = f \circ g_2$. One of the following statements is always true, and the other is not:
 - If f is one-to-one, then $g_1 = g_2$.
 - If f is onto, then $g_1 = g_2$.Prove the true statement, and give a counterexample to the false statement.
 - (b) Suppose $g_1 \circ f = g_2 \circ f$. One of the following statements is always true, and the other is not:
 - If f is one-to-one, then $g_1 = g_2$.
 - If f is onto, then $g_1 = g_2$.Prove the true statement, and give a counterexample to the false statement.

4. 16.17.

5. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be defined by $f(x) = x^2 - 8$. Find the set $f([-4, 7])$. Prove your answer.

6. 17.12.