

Math 108, problem set 05
Outline due: Wed Mar 09
Completed version due: Mon Mar 14
Last revision due: Wed Apr 27

Exercises (to be done but not turned in): 12.1, 12.4, 12.5, 12.6, 12.8, 12.9.

Problems to be turned in: All numbers refer to problems in the Yellow and Blue Book.

1. Let S be a nonempty subset of \mathbf{R} , and let U be an upper bound for S . Prove that the following are equivalent:
 - (Sup) $U = \sup S$.
 - (Arbitrarily close) For any $\epsilon > 0$, there exists some $x \in S$ such that $U - \epsilon < x$.

Suggestion: Instead of proving that (Arbitrarily close) implies (Sup), you may find it easier to prove the contrapositive.
2. Let S be the open interval $(-7, -2)$. Determine (guess) the value of $\sup S$, and prove that your guess is correct.
3. Let $S = \left\{ \frac{7n}{5n-2} \mid n \in \mathbf{Z}, n > 0 \right\}$. Determine (guess) the value of $\inf S$, and prove that your guess is correct.
4. Let $S = (2, \pi^2) \cap \mathbf{Q}$. Determine (guess) the value of $\sup S$, and prove that your guess is correct.
5. 12.8.
6. 12.13(a,b,c). (I.e., skip part (d).)
7. (a) Is it possible that there exists a real number M such that $r \leq M$ for all **rational** numbers r ? Prove or disprove.
(b) Is it possible that there exists a real number $m > 0$ such that $r \geq m$ for all **postive rational** numbers r ? Prove or disprove.