Math 108, problem set 04
Outline due: Wed Mar 02
Completed version due: Mon Mar 07
Last revision due: Mon Mar 21

Definitions:

block If $\{A_\alpha \mid \alpha \in I\}$ is a partition of a set X, we call the A_α the **blocks** of the partition.

Exercises (to be done but not turned in): 10.1, 10.2, 10.4, 10.5, 10.7, 11.2, 11.6.

Problems to be turned in: All numbers refer to problems in the Yellow and Blue Book.

1. 10.4.
2. 10.5.
3. Define a relation $\sim$ on $\mathbf{R}^2$ by saying that $(x_1, x_2) \sim (y_1, y_2)$ if and only if at least one of $x_1 - y_1$ and $x_2 - y_2$ is an integer. Determine whether or not this relation is reflexive, symmetric, or transitive. If a property holds, prove that it holds; if it does not, prove that it does not. If the relation is an equivalence relation, give the equivalence class of a general point.
4. 10.10.
5. Consider the relation $\sim$ on $\mathbf{R}^2$ defined by declaring that $(x, y) \sim (w, z)$ if and only if $2x + y = 2w + z$.
   (a) Prove that $\sim$ is an equivalence relation.
   (b) Give a geometric description/picture of a typical equivalence class under $\sim$. Also, describe special cases (non-typical equivalence classes), if any.
6. For $n \in \mathbf{Z}$, let $A_n = \{2n, 2n + 1\}$.
   (a) Draw a picture of $A_{-3}, \ldots, A_4$ on the real number line, and label each A_n.
   (b) Use the definition of partition to prove that $\mathcal{A} = \{A_n \mid n \in \mathbf{Z}\}$ is a partition of the set $\mathbf{Z}$. (You may use "K-12" facts that you know about even and odd numbers, division with remainder, etc.)
7. Let
   $$L_b = \left\{(x, y) \in \mathbf{R}^2 \mid y = x^2 + b\right\}.$$
   You are given that $\mathcal{A} = \{L_b \mid b \in \mathbf{R}\}$ is a partition of $\mathbf{R}^2$ (i.e., do not spend time checking that $\mathcal{A}$ really is a partition).
   (a) Define an equivalence relation $\sim$ whose equivalence classes are the blocks of $\mathcal{A}$. (Do not spend time checking that $\sim$ really is an equivalence relation.)
   (b) Prove that the equivalence classes under $\sim$ are precisely the blocks of $\mathcal{A}$. More precisely:
      i. Prove that for each $b \in \mathbf{R}$, there exists $(c, d) \in \mathbf{R}^2$ such that $E_{(c,d)} = L_b$; and
      ii. Prove that for each $(c, d) \in \mathbf{R}^2$, there exists $b \in \mathbf{R}$ such that $L_b = E_{(c,d)}$.