Math 108, problem set 03
Outline due: Wed Feb 17
Completed version due: Mon Feb 22
Last revision due: Mon Mar 21

Exercises (to be done but not turned in): 8.3-8.5, 8.8-8.10; 9.2-9.5, 9.8-9.9.
Problems to be turned in: All numbers refer to problems in the Yellow Book.

1. 8.7.

2. 8.9.

3. Let $I = \{x \in \mathbf{R} \mid 0 < x < 1\}$. For $x \in I$, let $A_x = (4 - x, 5 + x)$, i.e., let A_x be the open interval between $4 - x$ and $5 + x$. Prove that $\bigcap_{x \in I} A_x = [4, 5]$, the *closed* interval between 4 and 5 (inclusive).

4. Let A, B, C, and D be sets.

   (a) Find an example of sets A, B, C, and D where $A \times B \subseteq C \times D$ but $A \not\subseteq C$.

   (b) Complete and prove the following theorem: If $A \times B \subseteq C \times D$ and (extra condition on B), then $A \subseteq C$.

5. Let A, B, and C be sets.

   (a) Is $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$? Prove or give a counterexample.

   (b) Is $A \times (B \cup C) \supseteq (A \times B) \cup (A \times C)$? Prove or give a counterexample.

6. 9.8.